A Two-stage Adaptive Robust Model for Residential Micro-CHP Expansion Planning

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***Abstract*—This paper addresses the planning problem of residential micro combined heat and power (micro-CHP) systems (including micro-generation units, auxiliary boilers, and thermal storage tanks) considering the associated technical and economic factors. Since the accurate values of the thermal and electrical loads of this system cannot be exactly predicted for the planning horizon, the thermal and electrical load uncertainties are modeled using a two-stage adaptive robust optimization method based on a polyhedral uncertainty set. A solution method, which is composed of column-and-constraint generation (C&CG) algorithm and block coordinate descent (BCD) method, is proposed to efficiently solve this adaptive robust optimization model. Numerical results from a practical case study show the effective performance of the proposed adaptive robust model for residential micro-CHP planning and its solution method.**

***Index Terms*—Micro combined heat and power (micro-CHP) planning, two-stage adaptive robust optimization model, block coordinate descent method, polyhedral uncertainty set.**

NOMENCLATURE

A. Indices and Sets

Ψ^{FS}, Υ	Set of first-stage variables and associated feasible region
Ψ^{U}, Θ	Set of uncertain variables and polyhedral uncertainty set
Ψ^{SS}, Ξ	Set of second-stage variables and associated feasible region
Ψ^{MP}	Set of variables of master problem
Ψ^{P1}, Ψ^{P2}	Sets of variables of Problems 1 and 2 of block coordinate descent (BCD) method
j, g, u	Indices of available capacities for micro combined heat and power (micro-CHP) unit, boiler, and storage tank
k, l, n	Indices of available technologies for micro-CHP unit, boiler, and storage tank
r, r'	Iteration indices for column and constraint generation (C&CG) algorithm
s, s'	Scenario indices in out-of-sample analysis and benchmark stochastic model
t, d, h	Indices for years, representative days, and hours
v	Iteration index for block coordinate descent (BCD) method

B. Parameters

$\Delta L_{tdh}^{ele}, \Delta L_{tdh}^{Thermal}$	Variations of electrical and thermal loads with respect to their forecasting values
Γ	Uncertainty budget
α, β	Electrical and thermal efficiencies of micro-CHP unit
$\delta^{Resistor}$	Electricity-to-heat converting efficiency of electrical heating element
ρ_{tdh}^{ele}, ρ_{tdh}^{Gas}	Electricity consumption tariff and gas consumption tariff
ρ_{tdh}^{Sell}	Price of electricity sold to upstream grid
η	Efficiency of boiler
ξ	Heat loss coefficient
ε	Predefined tolerance parameter used for column-and-constraint generation (C&CG) algorithm convergence
Cc_k^{CHP}	Investment cost coefficient of micro-CHP unit with technology k
Cc_l^{Boiler}	Investment cost coefficient of boiler with technology l
Cc_n^{Tank}	Investment cost coefficient of storage tank with technology n
Cp_{kj}^{RatCHP}	Nominal capacity of micro-CHP unit with technology k and capacity j
$Cp_{lg}^{RatBoiler}$	Nominal capacity of boiler with technology l and capacity g
$Cp_{nu}^{RatTank}$	Nominal capacity of storage tank with technology n and capacity u
E^{MU}	The maximum capacity of electricity exchange with upstream grid
$\bar{E}^{Resistor}$	The maximum electric power of electrical heating element
$H^{MinTank}$	The lower bound of heat stored in storage tank
$H^{MCharge}$	The upper bound of storage tank for charging heat

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H^{MDis}	The upper bound of storage tank for discharging heat
I	Interest rate
$\overline{L}^{ele}_{tdh}$, $\overline{L}^{Thermal}_{tdh}$	Forecasting values of electrical and thermal loads
Mc^{RCHP}	Maintenance coefficient of micro-CHP unit
Mc^{RTank}	Maintenance coefficient of storage tank
$Mc^{RBoiler}$	Maintenance coefficient of boiler
N_d	Number of days represented by day d
NS	Number of scenarios in out-of-sample analysis
NS'	Number of scenarios in benchmark stochastic model
P_s	Probability of scenario s
THR	Heating ratio
UB, LB	Upper and lower bounds of problem
$\bar{w}_s$, $\bar{w}_{s'}$	Normalized probabilities of scenarios s and s'

C. Variables

γ	Auxiliary continuous modelling variable
$\lambda^{(v)}_{tdh}, \omega^{(v)}_{tdh}$	Dual variables associated with electrical and thermal loads in iteration v of BCD method
$\wedge$	Fixed value of variable
$b^{CHP}_{kj}, b^{Boiler}_{lg}$, b^{Tank}_{nu}	Binary investment variables of micro-CHP unit, boiler, and storage tank from different technologies/capacities
b^{Dis}_{tdh}	Binary variable indicating status of storage tank (1: discharging; 0: charging)
b^{u}_{tdh}	Binary variable indicating status of exchanging electricity with upstream grid (1: purchasing; 0: selling)
$C1^{(v)}$	Objective function value for Problem 1 of BCD method at iteration v
C^{InvCHP}	Investment cost of micro-CHP unit
$C^{InvBoiler}$	Investment cost of boiler
$C^{InvTank}$	Investment cost of storage tank
$C^{operation}_{s'}$	Operation cost of in-sample scenario s'
C^{OpeCHP}_{t}	Operation cost of micro-CHP unit in year t
$C^{OpeBoiler}_{t}$	Operation cost of boiler in year t
C^{Ele}_{t}	Cost of electricity purchased from upstream grid in year t
C^{MT}_{t}	Maintenance cost in year t
C^{Sell}_{t}	Revenue of selling electricity to upstream grid in year t
C^{SA}	Cost objective of benchmark stochastic model
E^{CHP}_{tdh}	Electricity produced by micro-CHP unit
$E^{Utility}_{tdh}$	Electricity purchased from grid
E^{Sell}_{tdh}	Electricity sold to grid
$E^{Resistor}_{tdh}$	Electricity consumed by electrical heating element
H^{CHP}_{tdh}	Heat produced by micro-CHP unit
H^{Boiler}_{tdh}	Heat produced by boiler
H^{Tank}_{tdh}	Heat stored in storage tank
H^{Charge}_{tdh}	Charged heat of storage tank
H^{Dis}_{tdh}	Discharged heat of storage tank
$H^{Resistor}_{tdh}$	Heat produced by electrical heating element
$L^{ele}_{tdh}, L^{Thermal}_{tdh}$	Uncertain electrical and thermal loads
$Z^{ele}_{tdh}, Z^{Thermal}_{tdh}$	Continuous modeling variables used to model L^{ele}_{tdh} and $L^{Thermal}_{tdh}$

I. Introduction

The micro combined heat and power (micro-CHP) systems have presented an effective solution for providing electrical and thermal energies for residential consumers. Micro-CHP systems can bring considerable economic benefits by recovering the heat wasted during the conversion of fossil fuels to electrical energy [1]. Micro-CHP systems are flexible, efficient, and reliable systems [2]. Therefore, they can be considered as a viable source of energy for residential buildings in the future smarter systems. However, the power and heat generated by micro-CHP systems have a reciprocal dependency, which determines the feasible operation region of these systems [3]. The optimal planning and scheduling of residential micro-CHP systems, considering their feasible operation regions, are essential to attain lower operation costs and higher energy efficiencies [4], [5]. Moreover, the reliable operation of these systems in the presence of uncertainty sources should be taken into account [6]. On the other hand, in a residential micro-CHP system, the thermal inertia of the thermal load is an important flexibility resource that can decrease the planning and operation cost [7]. In [7], the thermal inertia aggregation model (TIAM) has been proposed to model the thermal dynamic characteristics of the district heating network (DHN) and buildings. Simulation results have shown that TIAM can provide an accurate model of the DHN and buildings for the planning and operation of integrated energy systems, providing a basis for analyzing and evaluating the operation flexibility of DHNs.

The feasibility of micro-CHP installations in residential buildings requires a technical and economic viability study, which is closely related to the sizing of devices [8]. In [8], a linear programming model is developed to determine the design and sizing of the micro-CHP unit, auxiliary boiler, and thermal storage unit, considering the optimal operation strategy. The obtained results have shown that the optimal integration and sizing of the micro-CHP components considerably improve the economic, thermodynamic, and environmental results. The techno-economic assessment and optimization of Stirling engine micro-cogeneration systems in residential buildings have been addressed in [9]. By comparing the performance of various system configurations and different operation strategies, the optimal strategies for the integration of a Stirling-engine-based micro-cogeneration system into residential buildings have been determined. Detailed results of [9] have shown that an optimally operated micro-CHP system would result in a significant decrease in total cost, primary energy consumption, and CO_2 emissions compared with a conventional system. In [10], optimization models for the capacity and dispatch planning of residential micro-CHP systems have been developed. The numerical results of [10] have shown that considerable economic savings in annual costs can be obtained through an optimal sizing and operation of a system consisting of a micro-CHP unit, a backup boiler, and a storage tank.

For micro-CHP planning, the type and capacity of the micro-CHP components (including, e.g., micro-generation unit, auxiliary boiler, and thermal storage tank) should be selected considering the associated techno-economic aspects. Although the generation and transmission expansion planning of power system has recently attracted great attention in [11]-[14], to the best of the authors' knowledge, the generation expansion planning of residential micro-CHP systems has not been addressed in the previous literature.

The micro-CHP planning problem is subject to different uncertainty sources in the planning horizon, such as thermal and electrical load demand uncertainties. Therefore, it is necessary to develop non-deterministic micro-CHP planning frameworks to appropriately model these uncertainty sources. To address the impacts of the uncertainties in the planning horizon, stochastic programming (SP) and robust optimization (RO) methods can be used, which are standard tools to model uncertainties in different power system contexts [15]-[21]. SP methods are based on the probability distribution function (PDF) of uncertain variables. In [15], a stochastic optimization model has been proposed to jointly optimize the energy and reserve dispatches, and a set of scenarios has been generated to characterize the uncertainty of wind power. In [16], another SP method has been employed to model the uncertainties in distributed generation planning using scenario sampling. A multi-objective SP method has been proposed for joint energy and reserve market clearing in [17]. In this SP method, the uncertainties of unit and branch unavailability as well as the load forecasting uncertainties have been explicitly modeled using scenario trees.

In SP methods, a large number of scenarios are usually required to appropriately capture the uncertainty spectrum, which typically increases the computation burden of such uncertainty modeling methods. These scenarios, which may be generated from the PDF of uncertain variables, are intended to simulate the possible realizations of uncertainties. Therefore, the solution of SP is optimal on average for these in-sample scenarios. However, other out-of-sample realizations of uncertainties, which are unseen for the SP method, may occur in practice, and the SP method solution has no guarantee of optimality or even feasibility for these out-of-sample scenarios. In addition, gathering sufficient historical data to generate an adequate number of in-sample scenarios may not be an easy task in practice. Especially when the optimization problem involves multiple uncertainty sources, which is the case of residential micro-CHP planning problem, the aforementioned disadvantages can be highlighted.

On the other hand, RO methods are based on bounded intervals to model uncertain variables and do not require the exact PDF of uncertain variables. Thus, they need less historical data compared with SP methods. In addition, RO methods, which only consider the worst-case realization of uncertain variables, can provide a more tractable uncertainty modeling approach compared with SP methods. Besides, RO methods immunize the solution against the worst-case realization of the uncertain variables and thus immunize the solution against any realization of uncertain variables within the uncertainty set considered, while SP methods cannot guarantee the robustness of the solution.

However, this feature may lead to over-conservativeness of RO methods compared with SP methods. This over-conservativeness problem can be solved by adding a so-called degree of robustness to RO-based models [18]. Two-stage adaptive RO is an extension of RO to optimize both here-and-now decisions (which are made before the realization of uncertainties) and wait-and-see decisions (which are made after realization of uncertainties) [19]. An RO method has been presented in [20] for generation and transmission expansion planning of power system considering the uncertainties of estimated investment costs and forecasted electricity demand. An adaptive RO model for the expansion planning of a distribution system including distributed energy resources has been proposed in [21] considering the uncertainties of load demand and wind power.

The main contributions of this paper can be summarized as follows.

1) In this paper, a new expansion planning model for a residential micro-CHP system (consisting of a micro-generation unit, auxiliary boiler, and thermal storage tank) is proposed to meet the future heat and electricity demands. The proposed model is different from generation and transmission expansion planning models of power system, such as those presented in [11]-[14]. The reasons are that a residential micro-CHP system should supply both thermal and electrical loads (considering their coordination constraints), and its main components are different from those considered in generation and transmission expansion planning models of power system.

2) The proposed residential micro-CHP planning model is implemented in the form of a two-stage adaptive RO framework considering the uncertainties of thermal and electrical loads in the planning horizon. Moreover, to solve this two-stage adaptive RO problem, a solution method is proposed, which comprises the column-and-constraint generation (C&CG) and block coordinate descent (BCD) methods. Using the proposed solution method, this optimization problem is efficiently solved without entailing bilinear terms or linearization techniques.

The rest of this paper is organized as follows. In Section II, the uncertainty set is characterized. The proposed adaptive RO model for the micro-CHP planning problem is introduced in Section III. The proposed solution method is presented in Section IV. The numerical results obtained from the proposed model and solution method are provided in Section V. Finally, Section VI concludes the paper.

II. UNCERTAINTY CHARACTERIZATION

As shown in Fig. 1, a typical residential micro-CHP system consists of a micro-CHP unit, a backup boiler, and a thermal storage tank [22]. The micro-CHP unit usually uses natural gas as input fuel to generate electric power. The resultant high-temperature exhaust gases are utilized to supply the thermal loads. If the produced heat is higher than the thermal loads, the extra heat is stored in the thermal storage tank for later use. In contrast, if the produced heat cannot fully meet the thermal loads, the backup boiler and/or the heat stored in the thermal storage tank can be employed. Similarly, when the electric power generated by the micro-CHP unit exceeds the electric demand of consumer, additional electric power can be sold to the upstream grid or used to generate heat by an electrical heating resistor in the thermal storage tank. Moreover, the shortage of electricity can be compensated by purchasing electricity from the grid. Additionally, the electrical heating resistor can be used to convert some of the electricity purchased from the grid to heat, when the electricity price is low.

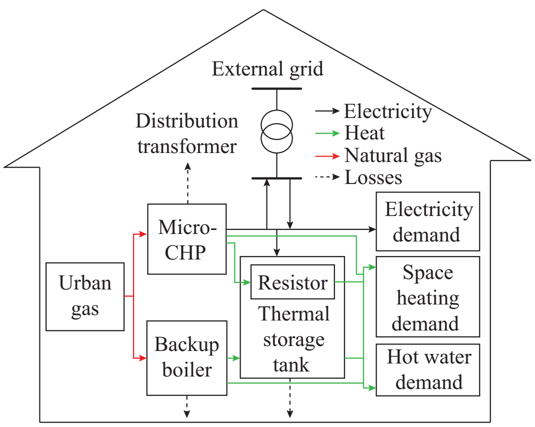

Fig. 1. Schematic representation of a typical residential micro-CHP system.

Nowadays, various technologies with different characteristics are available for micro-CHP units, such as internal combustion engines, Stirling engines, Rankine cycle generators, micro gas turbines, reciprocating engines, and fuel cells [2]. There are also a wide variety of technologies for boilers and storage tanks [23]-[26]. The selection of the best technologies for the residential micro-CHP system should be based on the techno-economic aspects. In addition, some uncertainty sources affect the investment decisions in the residential micro-CHP planning problem. In this paper, the uncertainties of thermal load (including space and water heating demand) and electrical load are considered. To characterize these uncertain variables, a polyhedral uncertainty set is defined based on their forecasting values and bounded intervals. The adaptive robust model proposed in this paper obtains a robust investment plan that can withstand against the worst-case realizations of the uncertain variables within the polyhedral uncertainty set, denoted as $\Theta = \{\Psi^U : (1)\text{-}(5)\}$.

$$L_{tdh}^{ele} = \overline{L}_{tdh}^{ele} + Z_{tdh}^{ele} \Delta L_{tdh}^{ele} \quad \forall t, \forall d, \forall h \tag{1}$$

$$L_{tdh}^{Thermal} = \overline{L}_{tdh}^{Thermal} + Z_{tdh}^{Thermal} \Delta L_{tdh}^{Thermal} \quad \forall t, \forall d, \forall h \tag{2}$$

$$Z_{tdh}^{ele} \in [0, 1] \quad \forall t, \forall d, \forall h \tag{3}$$

$$Z_{tdh}^{Thermal} \in [0, 1] \quad \forall t, \forall d, \forall h \tag{4}$$

$$\sum_h Z_{tdh}^{ele} + Z_{tdh}^{Thermal} \le \Gamma \quad \forall t, \forall d \tag{5}$$

To extract the worst-case realization of the uncertain variables within the polyhedral uncertainty set, thermal and electrical loads should increase as much as possible with respect to their forecasting values. Therefore, one-side bounded intervals for the uncertain variables have been considered in the uncertainty set Θ. In addition, unlike other adaptive RO methods, the proposed BCD method does not require binary modeling variables to construct the polyhedral uncertainty set and can directly work with the continuous modeling variables Z_{tdh}^{ele} and $Z_{tdh}^{Thermal}$ as indicated in (3) and (4) [27]. In the uncertainty set Θ, a pre-defined uncertainty budget in (5), i.e., Γ, has been used to control the robustness of the solution.

In the literature, for residential micro-CHP systems, the electricity purchasing price is usually considered based on the time-of-use (TOU) tariff [28]-[31]. Additionally, the natural gas purchasing price and electricity selling price for residential micro-CHP systems are considered based on tariffs [28]-[31]. For instance, this is the situation that we practically observe in Iran, where regulated prices based on tariffs are applied for electricity purchasing, natural gas purchasing, and electricity selling of residential micro-CHP systems. As these prices are defined based on tariffs, no uncertainty has been considered for them in the proposed model. However, if these prices do not follow tariffs and have uncertainty in a country, their uncertainties can be modeled similar to the load uncertainties. In other words, since electricity prices are continuous and uncertain variables, like the loads, we can model their uncertainties using bounded intervals in the polyhedral uncertainty set, as given in (1)-(5).

III. Tri-Level Adaptive Robust Model for Micro-CHP Planning

The proposed tri-level adaptive robust model for the planning of the residential micro-CHP system is formulated as:

$$\min_{\Psi^{FS} \in \Upsilon} (C^{InvCHP} + C^{InvBoiler} + C^{InvTank}) + \max_{\Psi^U \in \Theta} \min_{\Psi^{SS} \in \Xi} \left[\sum_t \frac{1}{(1+I)^t} (C_t^{OpeCHP} + C_t^{OpeBoiler} + C_t^{Ele} + C_t^{MT} - C_t^{Sell}) \right] \tag{6}$$

The above min-max-min problem minimizes the worst-case total cost of the residential micro-CHP system throughout the planning period, which consists of the investment costs (C^{InvCHP}, $C^{InvBoiler}$, and $C^{InvTank}$), the present operation costs (C_t^{OpeCHP}, $C_t^{OpeBoiler}$, C_t^{Ele}, and C_t^{MT}), and the negative value of the present benefit obtained by selling electricity to the grid (C_t^{Sell}). In the following subsections, these three optimization levels are introduced.

A. The First Level

The first level of the adaptive robust model in (6) determines the first-stage investment decisions, i.e., $\Psi^{FS} = \{\{b_{kj}^{CHP}\}_{\forall k, \forall j}, \{b_{lg}^{Boiler}\}_{\forall l, \forall g}, \{b_{nu}^{Tank}\}_{\forall n, \forall u}\}$. The feasible region Υ at the first level is formulated as $\Upsilon = \{\Psi^{FS} : (7)\text{-}(13)\}$.

$$\begin{cases} \sum_k \sum_j b_{kj}^{CHP} \le 1 \\ \sum_l \sum_g b_{lg}^{Boiler} \le 1 \\ \sum_n \sum_u b_{nu}^{Tank} \le 1 \end{cases} \tag{7}$$

$$C^{InvCHP} = \sum_k \sum_j (Cc_k^{CHP} \cdot Cp_{kj}^{RatCHP} \cdot b_{kj}^{CHP}) \tag{8}$$

$$C^{InvBoiler} = \sum_l \sum_g (Cc_l^{Boiler} \cdot Cp_{lg}^{RatBoiler} \cdot b_{lg}^{Boiler}) \tag{9}$$

$$C^{InvTank} = \sum_n \sum_u (Cc_n^{Tank} \cdot Cp_{nu}^{RatTank} \cdot b_{nu}^{Tank}) \tag{10}$$

$$b_{kj}^{CHP} \in \{0,1\} \quad \forall k, \forall j \tag{11}$$

$$b_{lg}^{Boiler} \in \{0,1\} \quad \forall l, \forall g \tag{12}$$

$$b_{nu}^{Tank} \in \{0,1\} \quad \forall n, \forall u \tag{13}$$

Constraint (7) limits the number of installations of the micro-CHP unit, auxiliary boiler, and storage tank. Here, one installation has been considered for each component in the planning horizon. However, any other number of installations can be considered based on the planner's preferences. Constraints (8)-(10) define the investment costs to be included in the first level of the tri-level adaptive robust model (6). Constraints (11)-(13) represent the binary investment decision variables.

B. The Second Level

The second level of the adaptive robust model in (6) determines $\Psi^U = \{\{Z_{tdh}^{ele}, Z_{tdh}^{Thermal}\}_{\forall t, \forall d, \forall h}\}$ to extract the worst-case realization of the uncertain variables. The feasible region for this level, i.e., the polyhedral uncertainty set Θ, has already been defined in Section II.

C. The Third Level

The third level of the adaptive robust model in (6) determines the second-stage operation decisions, i.e., $\Psi^{SS} = \{\{b_{tdh}^u\}, \{E_{tdh}^{CHP}\}, \{H_{tdh}^{CHP}\}, \{H_{tdh}^{Boiler}\}, \{H_{tdh}^{Tank}\}, \{E_{tdh}^{Utility}\}, \{b_{tdh}^{Dis}\}, \{H_{tdh}^{Charge}\}, \{H_{tdh}^{Dis}\}, \{E_{tdh}^{Sell}\}, \{E_{tdh}^{Resistor}\}, \{H_{tdh}^{Resistor}\}\}_{\forall t, \forall d, \forall h}$. The feasible region Ξ for the second-stage variables can be described as $\Xi = \{\Psi^{SS}: (14)\text{-}(31)\}$.

$$0 \le E_{tdh}^{CHP} \le \sum_k \sum_j Cp_{kj}^{RatCHP} \cdot b_{kj}^{CHP} \quad \forall t, \forall d, \forall h \tag{14}$$

$$0 \le H_{tdh}^{Boiler} \le \sum_l \sum_g Cp_{lg}^{RatBoiler} \cdot b_{lg}^{Boiler} \quad \forall t, \forall d, \forall h \tag{15}$$

$$\sum_n \sum_u H^{MinTank} \cdot b_{nu}^{Tank} \le H_{tdh}^{Tank} \le \sum_n \sum_u Cp_{nu}^{RatTank} \cdot b_{nu}^{Tank} \quad \forall t, \forall d, \forall h \tag{16}$$

$$0 \le H_{tdh}^{CHP} \le \frac{\beta E_{tdh}^{CHP}}{\alpha} \quad \forall t, \forall d, \forall h \tag{17}$$

$$0 \le E_{tdh}^{Resistor} \le \bar{E}^{Resistor} \quad \forall t, \forall d, \forall h \tag{18}$$

$$H_{tdh}^{Resistor} = \delta^{Resistor} E_{tdh}^{Resistor} \quad \forall t, \forall d, \forall h \tag{19}$$

$$E_{tdh}^{CHP} + E_{tdh}^{Utility} - E_{tdh}^{Sell} - E_{tdh}^{Resistor} = L_{tdh}^{ele} \quad \forall t, \forall d, \forall h \tag{20}$$

$$0 \le E_{tdh}^{Utility} \le E^{MU} b_{tdh}^u \quad \forall t, \forall d, \forall h \tag{21}$$

$$0 \le E_{tdh}^{Sell} \le E^{MU} (1 - b_{tdh}^u) \quad \forall t, \forall d, \forall h \tag{22}$$

$$H_{tdh}^{CHP} + H_{tdh}^{Boiler} + H_{tdh}^{Resistor} + H_{tdh}^{Dis} - H_{tdh}^{Charge} = L_{tdh}^{Thermal} \quad \forall t, \forall d, \forall h \tag{23}$$

$$0 \le H_{tdh}^{Dis} \le H^{MDis} b_{tdh}^{Dis} \quad \forall t, \forall d, \forall h \tag{24}$$

$$0 \le H_{tdh}^{Charge} \le H^{MCharge} (1 - b_{tdh}^{Dis}) \quad \forall t, \forall d, \forall h \tag{25}$$

$$H_{t,d,(h+1)}^{Tank} = (1-\xi) H_{t,d,h}^{Tank} + H_{t,d,h}^{Charge} - H_{t,d,h}^{Dis} \quad \forall t, \forall d, \forall h \text{ if } h \ne 24 \tag{26}$$

$$C_t^{OpeCHP} = \sum_d \sum_h N_d E_{tdh}^{CHP} \frac{\rho_{tdh}^{Gas}}{\alpha \cdot THR} \quad \forall t \tag{27}$$

$$C_t^{OpeBoiler} = \sum_d \sum_h N_d H_{tdh}^{Boiler} \frac{\rho_{tdh}^{Gas}}{\eta \cdot THR} \quad \forall t \tag{28}$$

$$C_t^{MT} = \sum_d \sum_h N_d (E_{tdh}^{CHP} \cdot Mc^{RCHP} + H_{tdh}^{Boiler} \cdot Mc^{RBoiler} + H_{tdh}^{Tank} \cdot Mc^{RTank}) \quad \forall t \tag{29}$$

$$C_t^{Ele} = \sum_d \sum_h N_d E_{tdh}^{Utiility} \rho_{tdh}^{ele} \quad \forall t \tag{30}$$

$$C_t^{Sell} = \sum_d \sum_h N_d E_{tdh}^{Sell} \rho_{tdh}^{Sell} \quad \forall t \tag{31}$$

In (14)-(16), E_{tdh}^{CHP}, H_{tdh}^{Boiler}, and H_{tdh}^{Tank} are limited based on the capacities selected at the first level. The generated heat of the micro-CHP unit is restricted in (17). The electricity consumed by the electrical heating element is limited in (18). The heat produced by the electrical heating element is calculated in (19). Constraint (20) represents the electric power balance in the system. Constraints (21) and (22) limit the electricity purchased from/sold to the grid, respectively. The binary variable b_{tdh}^u in (21) and (22) avoids simultaneous purchasing and selling of electricity. The heat balance constraint of the system is given in (23). Similarly, (24) and (25) limit the discharging and charging of heat in the storage tank. The binary variable b_{tdh}^{Dis} in (24) and (25) avoids simultaneous discharging and charging of heat. Constraint (26) relates the heat stored in each hour to the stored heat (considering heat loss coefficient ξ), charged heat, and discharged heat in the previous hour. For the first hour of each day, the previous hour is considered as the last hour of the previous day. The annual operation costs of the micro-CHP unit and the auxiliary boiler are calculated in (27) and (28), respectively. The annual maintenance cost of the components of the residential micro-CHP system is calculated in (29). The annual cost/revenue of purchasing/selling electricity from/to the upstream grid are given in (30) and (31), respectively.

IV. Solution Methodology

For solving the proposed tri-level adaptive robust model presented in the previous section, it is first decomposed to a "min" master problem and a "max-min" sub-problem using C&CG algorithm. The master problem at iteration r of the C&CG algorithm is formulated as:

$$\begin{cases} \min_{\Psi^{MP}} (C^{InvCHP} + C^{InvBoiler} + C^{InvTank}) + \gamma \\ \text{s.t. } (7)\text{-}(13), (33)\text{-}(35) \end{cases} \tag{32}$$

$$\gamma \ge 0 \tag{33}$$

$$\gamma \geq \sum_t \frac{1}{(1+I)^t} (C_t^{OpeCHP(r')} + C_t^{OpeBoiler(r')} + C_t^{Ele(r')} + C_t^{MT(r')} - C_t^{Sell(r')}) \quad r' = 1, 2, ..., r-1 \tag{34}$$

$$(14)\text{-}(31): r' = 1, 2, ..., r-1 \tag{35}$$

The master problem consists of constraints (7)-(13) pertaining to the first-stage investment decisions and a set of primal cuts (33)-(35) added to the master problem at each iteration of the C&CG algorithm. In other words, at each iteration, the set of primal cuts (33)-(35) is added to the master problem based on the worst-case realization of uncertain variables obtained from the sub-problem in the previous iterations, which is denoted by $\hat{L}_{thd}^{ele(r')}$ and $\hat{L}_{tdh}^{Thermal(r')}$, where $r' = 1, 2, ..., r-1$. In the first iteration, no primal cut is added to the master problem. The decision variables of the master problem include the first-stage decision variables and also the variables used to construct primal cuts as: $\Psi^{MP} = \{\Psi^{FS}, \delta, \{\{b_{tdh}^{u(r')}\}, \{E_{tdh}^{Sell(r')}\}, \{E_{tdh}^{Utility(r')}\}, \{H_{tdh}^{Dis(r')}\}, \{b_{tdh}^{Dis(r')}\}, \{E_{tdh}^{CHP(r')}\}, \{H_{tdh}^{Boiler(r')}\}, \{H_{tdh}^{Tank(r')}\}, \{H_{tdh}^{Charge(r')}\}, \{E_{tdh}^{Resistor(r')}\}\}_{\forall t, \forall d, \forall h, r' = 1, 2, ..., r-1}\}$.

After solving the master problem, its results for the first-stage decisions, i. e., Ψ^{FS}, are sent to the sub-problem, as shown in the flowchart in Fig. 2.

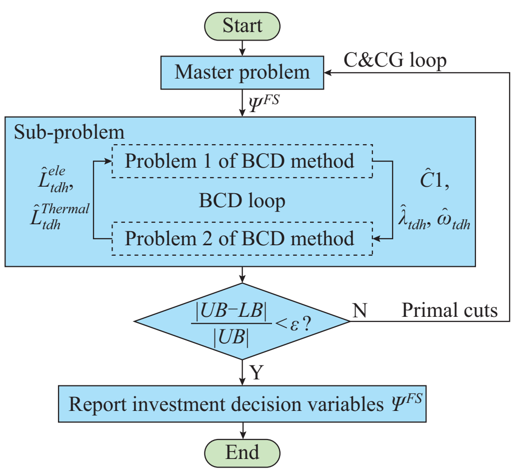

Fig. 2. Flowchart of proposed solution method.

The second step in solving the proposed tri-level adaptive robust model is to extract the worst-case realization of uncertain variables in the sub-problem for the obtained first-stage investment decisions (which are shown by $\hat{b}_{kj}^{CHP}$, $\hat{b}_{lg}^{Boiler}$, and $\hat{b}_{nu}^{Tank}$). Unlike the single-level master problem, the sub-problem is a bi-level optimization problem. In this paper, the BCD method is used to efficiently solve the sub-problem. In the BCD method, the bi-level sub-problem is further decomposed into Problem 1 for the operation variables, where the uncertain variables are fixed, and Problem 2 for the uncertain variables, where the first-order Taylor series is used to represent the cost objective function with respect to the uncertain variables. Problems 1 and 2 of the BCD method are solved iteratively until the BCD method converges. In fact, the BCD loop is inside the outer loop related to the C&CG algorithm, as shown in Fig. 2.

1) Problem 1 of the BCD method: given the first-stage investment decisions $\hat{b}_{kj}^{CHP}$, $\hat{b}_{lg}^{Boiler}$, and $\hat{b}_{nu}^{Tank}$ obtained from the master problem, and the values of the uncertain variables $\hat{L}_{tdh}^{ele(v-1)}$ and $\hat{L}_{tdh}^{Thermal(v-1)}$ obtained from the previous iteration $v-1$ of the BCD method, Problem 1 at iteration v is formulated as:

$$\begin{cases} C1^{(v)} = \min_{\Psi^{P1}} \sum_t \frac{1}{(1+I)^t} (C_t^{OpeCHP} + C_t^{OpeBoiler} + C_t^{Ele} + C_t^{MT} - C_t^{Sell}) \\ \text{s.t. } (14)\text{-}(31) \end{cases} \tag{36}$$

$$L_{tdh}^{ele} = \hat{L}_{tdh}^{ele(v-1)} : \lambda_{tdh}^{(v)} \quad \forall t, \forall d, \forall h \tag{37}$$

$$L_{tdh}^{Thermal} = \hat{L}_{tdh}^{Thermal(v-1)} : \omega_{tdh}^{(v)} \quad \forall t, \forall d, \forall h \tag{38}$$

where $\Psi^{P1} = \{\Psi^{SS}, \{L_{tdh}^{ele}, L_{tdh}^{Thermal}\}_{\forall t, \forall d, \forall h}\}$. The dual variables $\lambda_{tdh}^{(v)}$ and $\omega_{tdh}^{(v)}$ indicate the sensitivity of the objective function $C1^{(v)}$ with respect to uncertainties L_{dht}^{ele} and $L_{dht}^{Thermal}$ in iteration v of the BCD method, respectively. These dual variables are used in Problem 2 of the BCD method for constructing the first-order expansion of the Taylor series associated with the objective function to obtain the worst-case realization of the uncertain variables.

2) Problem 2 of the BCD method: given $\hat{C}1^{(v)}$, $\hat{\lambda}_{tdh}^{(v)}$, and $\hat{\omega}_{tdh}^{(v)}$ obtained from the solution of Problem 1, and also $\hat{L}_{tdh}^{ele(v-1)}$ and $\hat{L}_{tdh}^{Thermal(v-1)}$ obtained from the previous iteration $v-1$ of the BCD method, Problem 2 is formulated as:

$$\begin{cases} \max_{\Psi^{P2}} \left\{ \hat{C}1^{(v)} + \sum_t \sum_d \sum_h [\hat{\lambda}_{tdh}^{(v)} (L_{tdh}^{ele} - \hat{L}_{tdh}^{ele(v-1)}) + \hat{\omega}_{tdh}^{(v)} (L_{tdh}^{Thermal} - \hat{L}_{tdh}^{Thermal(v-1)})] \right\} \\ \text{s.t. } (1)\text{-}(5) \end{cases} \tag{39}$$

where $\Psi^{P2} = \{\Psi^U, \{L_{tdh}^{ele}, L_{tdh}^{Thermal}\}_{\forall t, \forall d, \forall h}\}$. Problem 1 for operation variables and Problem 2 for uncertain variables are related to the third and second levels of the proposed tri-level adaptive robust model, respectively. Considering the forecasting values $\overline{L}_{dht}^{ele}$ and $\overline{L}_{dht}^{Thermal}$ as the initial values for uncertain variables, the BCD method starts to iteratively solve Problems 1 and 2 until the value of $C1^{(v)}$ remains within a pre-defined interval in two successive iterations.

In the C&CG algorithm, the master problem and sub-problem provide the lower and upper bounds for the problem. As shown in Fig. 2, the C&CG algorithm terminates if UB and LB values are within a predetermined tolerance.

The nested C&CG algorithm devised in [32] can also be used to solve tri-level adaptive robust models with binary variables at the second stage of the problem. However, this method needs the transformation of the max-min problem at the second stage to an equivalent single-level optimization problem. The resulting single-level sub-problem will be a bilinear mathematical programming problem including non-convex product terms of the middle-level uncertain variables and the lower-level dual variables. These bilinear terms need to be linearized at the expense of introducing additional binary variables pertaining to the polyhedral uncertainty set [33]. More importantly, to perform this linearization, we need to set the lower and upper limits of the dual variables as the big-M values for the linearization [34]. Assigning either too low or too high values for the lower and upper limits of the dual variables can lead to low-quality solutions and even convergence problems. Therefore, assigning appropriate big-M values can be a challenging task. On the other hand, the proposed BCD-based solution method does not have dualization or subsequent linearization, and, therefore, does not require setting the big-M values for the lower and upper limits of the dual variables [27], [34]. In addition, unlike the nested C&CG algorithm that uses binary modeling variables to construct polyhedral uncertainty sets (since these binary auxiliary variables are required by the disjunctive programming and linearization techniques), the BCD-based method uses the real continuous form of uncertainties in the polyhedral uncertainty set as indicated in (3) and (4). Thus, the BCD-based method can efficiently solve the proposed adaptive robust micro-CHP planning model as shown in the next section.

V. Simulation Results

In this section, the proposed tri-level adaptive robust model for micro-CHP planning is tested on a residential micro-CHP system with the structure shown in Fig. 1. Four different types including internal combustion engine, Stirling engine, fuel cell, and Rankine cycle engine are considered as the planning options for the micro-CHP unit. The techno-economic data of these types can be found in [23]-[26]. In addition, two boiler types (including conventional boiler and system boiler) and two storage tank types (including Heat-Flo storage tank and Hydroflex storage tank) are considered as the boiler and storage tank planning options with the technical and economic data given in [35]-[37]. For each type of each micro-CHP component, 20 different capacities are assumed. The proposed model should also select the optimum capacity of each component. Thus, there are totally $(4\times20)\times(2\times20)\times(2\times20)=1.28\times10^5$ planning options for this residential micro-CHP test system. This large solution space illustrates the importance of the proposed model to optimize the residential micro-CHP expansion plan. Other techno-economic data of micro-CHP components, such as their electrical and thermal efficiencies, can be found in [22], [23], and [38]. In addition, the planning horizon is five years and the annual interest rate is 10%.

The profile of hourly electrical and thermal loads during one year has been constructed using Design-Builder software [39]. Also, a 5% annual growth rate is assumed for the loads. In this study, four representative days are selected for each year using k-means clustering technique [40]. Typically, the number of operation conditions considered increases with the number of representative days, which leads to a more accurate data modeling resulting in a lower cost, but at the expense of a higher computation burden [21]. The forecasting values of hourly electrical and thermal loads for the representative day in different seasons of the first year are shown in Fig. 3. Moreover, the electricity purchasing price has been considered based on TOU tariff as shown in Table I [28]. The natural gas purchasing price and electricity selling price are assumed to be 0.273 \$/m^3 and 0.0897 \$/kW, respectively, for the first year of the planning horizon [28]. Also, it is assumed that natural gas, electricity purchasing, and electricity selling prices have a 5% increase in each of the next years. The power exchange of the residential micro-CHP system with the upstream grid is limited to 35 kW [28].

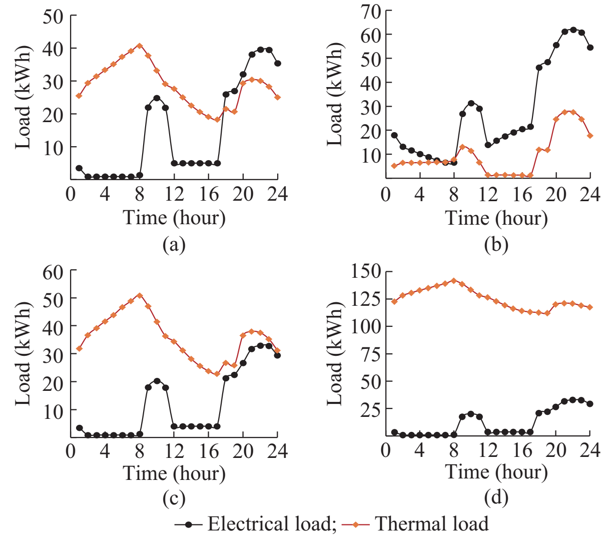

Fig. 3. Forecasting values of hourly electrical and thermal loads for representative day in different seasons. (a) Spring. (b) Summer. (c) Fall. (d) Winter.

TABLE I
TOU Tariff for Electricity Price

TOU tariff	Price (\$/kWh)	Hours in a day (warm seasons)	Hours in a day (cold seasons)
Off-peak	0.0442	23:00-07:00	22:00-05:00
Mid-peak	0.0866	08:00-19:00	06:00-17:00
On-peak	0.2461	20:00-22:00	18:00-21:00

A. Optimal Solution Versus Degree of Robustness

Given two different types of loads (electrical and thermal) and 24 operation hours in each representative day, the uncertainty budget Γ in (5) can adopt different values between 0 and $24\times2=48$. The results obtained from the proposed adaptive robust micro-CHP planning model for different values of Γ are shown in Table II.

TABLE II
Optimal Solution Versus Uncertainty Budget

Uncertainty budget Γ	Model cost (\$)	Micro-CHP unit capacity (kW)	Boiler capacity (kW)	Storage tank capacity (kW)
0	301352.4	44	60	10
12	318741.6	47	70	16
24	328007.8	48	73	17
36	340628.5	50	76	19
48	346381.7	51	77	21

According to Table II, by increasing Γ from 0 to 48, the total cost of the proposed adaptive robust micro-CHP planning model increases. The lowest cost is obtained by $\Gamma=0$ when none of the uncertain variables can deviate from its forecasting value. In fact, the uncertainty of electrical and thermal loads is not considered when the uncertainty budget equals 0 (similar to a deterministic model that does not consider uncertainty sources in its decision-making process). By increasing Γ from 0, a number of uncertain variables can deviate from their forecasting values and adopt their worst-case realizations within the polyhedral uncertainty set defined in (1)-(5). Thus, the proposed model can provide robustness against these uncertain variables. A higher value of Γ leads to considering a higher number of uncertain variables, which leads to a higher robustness level of the proposed model, but at the expense of a higher model's cost. In other words, this higher robustness against uncertainties comes at a cost. As Table II illustrates, by increasing the budget of uncertainty, the optimal capacities of the micro-CHP unit, boiler, and storage tank increase to supply higher electrical and thermal loads. As described in Section II, the worst-case realization of the uncertain variables includes the highest electrical and thermal load values based on the value of Γ, within the polyhedral uncertainty set.

However, from the results of the model's cost, as shown in Table II, we cannot observe the robustness worth of each case and select the best investment decisions for the micro-CHP planning. Therefore, an ex-post out-of-sample analysis is required to truly evaluate the performance of each case against unseen out-of-sample scenarios and to fine-tune the uncertainty budget in the proposed tri-level adaptive robust model for micro-CHP planning.

B. Evaluating Solutions Using Out-of-sample Analysis

The out-of-sample analysis can appropriately assess the robustness worth, in addition to the robustness cost, of different cases in Section V-A with different uncertainty budgets. To perform the out-of-sample analysis, 1000 various unseen scenarios are first generated for the uncertain electrical and thermal loads. These sample scenarios are generated using the normal probability distribution function and unseen for all models of Table IV to truly evaluate their performance. To model the uncertainty of hourly load forecasting in the micro-CHP planning, the continuous normal PDF has been divided into 7 discrete intervals [41]. The mid-value of each interval represents that interval. The probability of each out-of-sample scenario has been normalized as:

$$\bar{w}_s = \frac{P_s}{\sum_{s=1}^{NS} P_s} \quad s = 1, 2, ..., NS \tag{40}$$

The sum of $\bar{w}_s$ values for all out-of-sample scenarios is equal to one.

Since the out-of-sample scenarios are unseen, i. e., they have not been considered in the proposed model, the performance of each case in Table II can be truly evaluated by these scenarios. To evaluate each case by the out-of-sample analysis, the first-stage investment decision variables are fixed on the investment results obtained from that case, and the second-stage operation decision variables are optimized for each out-of-sample scenario considering its realized uncertain variables. Finally, the costs of all out-of-sample scenarios are aggregated based on their normalized probabilities to obtain the out-of-sample cost for the considered case. More details of the out-of-sample analysis can be found in [27], [42].

The results of the out-of-sample analysis for different cases of Table II are presented in Table III. Table III shows that the most conservative case (with the highest Γ value) of the proposed tri-level adaptive robust micro-CHP planning model is not necessarily the best solution from the perspective of the out-of-sample analysis. Indeed, as shown in Table III, $\Gamma = 12$ leads to the best solution with the lowest out-of-sample cost. In fact, the cases with the higher values of Γ are excessively conservative. A high robustness cost, which is required for installing a higher capacity of the micro-CHP unit, boiler, and storage tank as illustrated in Table II, is incurred in these over-conservative cases to be immunized against less likely realizations of the uncertainties, while a low robustness worth is obtained in return. The robustness worth is reflected by decreasing the operation cost of the proposed adaptive robust model, such as decreasing the cost of extra energy purchased from the upstream grid encountering various realizations of uncertainties. However, the robustness worth obtained from low-probability scenarios is low as these scenarios have a minor impact on the out-of-sample cost. Therefore, these over-conservative cases considering both the robustness cost and the robustness worth lead to higher out-of-sample costs than the case with the optimal uncertainty budget, i.e., $\Gamma = 12$. On the contrary, the case with $\Gamma = 0$ is an under-conservative case as it cannot provide any robustness against the uncertainties leading to a high out-of-sample cost. Therefore, the fine-tuned uncertainty budget of the proposed adaptive robust model for this residential micro-CHP planning test case is $\Gamma = 12$, which finds an appropriate compromise between the robustness cost and the robustness worth.

TABLE III
OUT-OF-SAMPLE COST FOR DIFFERENT UNCERTAINTY BUDGET VALUES

Uncertainty budget Γ	Out-of-sample cost ($)
0	379448.89
12	361047.49
24	379582.21
36	381043.15
48	382718.22

C. Comparison with Deterministic and Stochastic Models

In this section, the results of the proposed adaptive robust micro-CHP planning model are compared with the results of deterministic and stochastic micro-CHP planning models in the test case of residential micro-CHP. This comparison is carried out using out-of-sample analysis, and its results for different planning models are given in Table IV. The compact formulation of the employed stochastic micro-CHP planning model is given in Appendix A.

As shown in Table IV, while the deterministic micro-CHP planning model has the lowest model's cost (reported in the second column), it obtains the highest out-of-sample cost (reported in the third column). Thus, the deterministic model has the poorest performance encountering unseen out-of-sample scenarios. This is because the deterministic model completely ignores the uncertainty sources of the micro-CHP planning, and only considers the forecasting values of uncertain thermal and electrical loads. Thus, its model's cost is unrealistically low, as other realizations of uncertain variables (other than the forecasting values) can occur in practice, which are not considered in the deterministic model. Therefore, the deterministic model shows a poor performance encountering these realizations, leading to its highest out-of-sample cost in Table IV. Table IV shows that the stochastic micro-CHP planning model has somewhat better out-of-sample performance than the deterministic micro-CHP planning model as it considers some different realizations of uncertain variables using its in-sample scenarios. As shown in Table IV, the proposed adaptive robust planning model has the best out-of-sample performance with the lowest out-of-sample cost, since the proposed model not only considers the uncertain variables but also provides appropriate immunization against them.

TABLE IV
COMPARISON OF PROPOSED ADAPTIVE ROBUST MODEL WITH DETERMINISTIC AND STOCHASTIC MODELS

Model	Model's cost ($)	Out-of-sample cost ($)	Investment cost ($)	Out-of-sample operation cost ($)	Computation time (s)
Deterministic	301352.41	379448.89	212375	167074	212
Stochastic	303745.80	377322.25	218400	158922	505
Proposed	318741.62	361047.49	228530	132517	238

To better explain these comparative results, the two main components of the out-of-sample cost including the investment cost (which is the total investment cost of the micro-CHP unit, boiler, and storage tank of the micro-CHP system) and the out-of-sample operation cost (which is the aggregated operation cost of all out-of-sample scenarios considering their normalized probabilities) are reported for different planning models in the fourth and fifth columns of Table IV. These results indicate that the proposed tri-level adaptive robust model with a slightly higher investment cost compared with the deterministic and stochastic planning models (which is incurred to provide robustness against uncertain variables) leads to a significantly lower operation cost encountering unseen out-of-sample scenarios. Thus, the proposed planning model results in a lower out-of-sample cost (which is the sum of the investment cost and the out-of-sample operation cost) compared with the deterministic and stochastic planning models.

The computation time of the deterministic, stochastic, and proposed models is also presented in Table IV. It can be observed that the computation time of the proposed adaptive robust model on the residential micro-CHP test system is around 4 minutes, which is lower than the computation time of the stochastic model and is close to that of the deterministic model. The low computation time of the proposed tri-level adaptive robust model indicates its high computational efficiency. In addition, it is worthwhile to note that the proposed tri-level adaptive robust model is for the residential micro-CHP expansion planning with a five-year planning horizon, and thus, a 4-minute computation time is completely reasonable.

To statistically validate the results of the out-of-sample analysis, the convergence coefficient [42] has been calculated for each model in Table IV. All the convergence coefficients of the out-of-sample analysis are well below 0.01, which means that the out-of-sample analysis has sufficiently converged for all models [42].

The results of the electricity/heat generated by the micro-CHP unit and the heat generated by the boiler obtained from the proposed model with $\Gamma=12$ are shown in Fig. 4 on the representative day in winter season of the first year of the planning horizon. Also, the results obtained from this model for the electricity purchased from/sold to the upstream grid on the representative day in fall season in the first year are illustrated in Fig. 5.

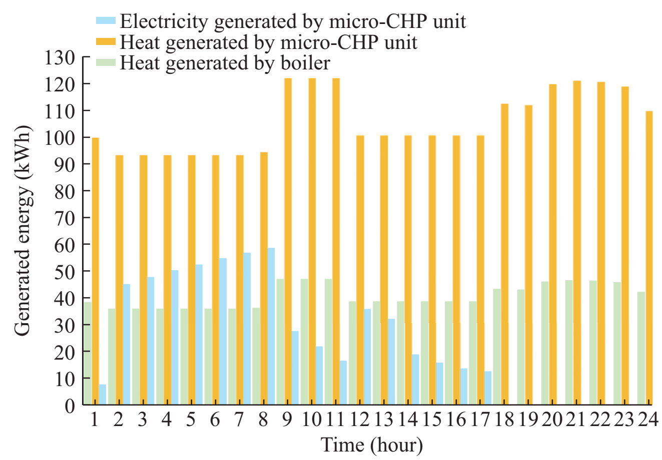

Fig. 4. Generated electricity and heat of micro-CHP unit and generated heat of boiler on representative day in winter season of the first year.

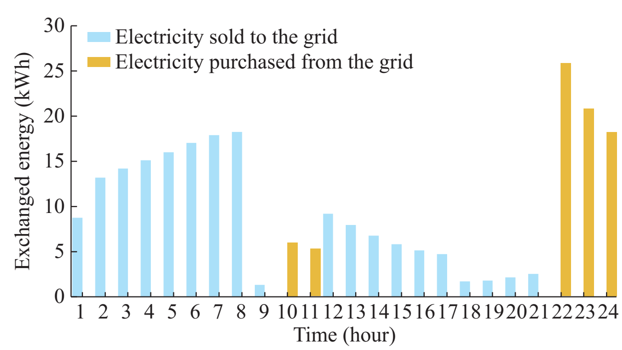

Fig. 5. Electricity purchased from/sold to upstream grid on representative day in fall season of the first year.

From Fig. 4 and the hourly forecasted loads on the representative day in winter season illustrated in Fig. 3(d), it is seen that both the micro-CHP unit and the boiler generate heat in hours 1-17 with high heat load values. Moreover, in hours 1-17 with low electricity load values, the excess electricity generation of the micro-CHP unit can be sold to the grid. Compared with these hours, the next hours 18-24 have lower thermal load values and higher electric load values. The micro-CHP unit supplies the electric loads of these hours, and, at the same time, its thermal generation supplies the lower thermal loads of these hours. Thus, the heat generation of the boiler is not needed in hours 18-24.

Figure 5 shows that the micro-CHP system purchases electricity from the upstream grid at off-peak hours 22-24 with low electricity purchasing prices (illustrated in Table I) and high electric load values (illustrated in Fig. 3(c)). Lower electricity purchase values are in the mid-peak hours 10 and 11 with higher electricity purchasing prices. In other hours with lower electric load values, the micro-CHP system can sell electricity to the grid as shown in Fig. 5.

In this paper, all simulations have been run using CPLEX solver within the GAMS software package [43] on a Core i5 2.5 GHz computer with 6 GB of RAM. In this paper, the relative duality gap is set to be 0.001%. The computation time of the proposed adaptive robust micro-CHP planning model on the residential micro-CHP test system is around 4 minutes. This short computation time shows the high computational efficiency of the proposed model.

VI. Conclusion

In this paper, a two-stage adaptive robust model has been proposed for residential micro-CHP planning. The uncertainty sources of thermal and electric loads have been modeled in this paper. C&CG algorithm and BCD method are used to solve the proposed model. The proposed model and solution method have been tested on a residential micro-CHP test system. The results have shown that a higher uncertainty budget in the proposed adaptive robust micro-CHP planning model leads to a higher capacity of the micro-CHP unit, boiler, and storage tank resulting in a higher investment cost and a higher model's cost. In fact, a higher budget of uncertainty provides a more robust solution, but at a higher robustness cost. To properly evaluate each robustness level and select the best investment decision for the planning problem, an ex-post out-of-sample analysis using various unseen scenarios of thermal and electric loads has been performed. Using the out-of-sample analysis, the robustness worth of each robustness level, which is reflected as decreasing the operation cost encountering unseen realizations of uncertainties, can be evaluated in addition to the robustness cost. Thus, with the aid of the out-of-sample analysis, the robustness level of the proposed model can be fine-tuned leading to an appropriate compromise between the robustness cost and the robustness worth. Furthermore, it has been shown that the proposed adaptive robust micro-CHP planning model outperforms deterministic and stochastic micro-CHP planning models in the out-of-sample analysis since it can provide appropriate immunization against the uncertainties.

Appendix A

Appendix A presents the employed stochastic micro-CHP planning model. To implement the model, at first, 1000 various scenarios have been generated for the uncertain electric and thermal loads. The procedure of generating in-sample scenarios for the stochastic model is the same as the procedure of generating out-of-sample scenarios for the out-of-sample analysis, as described in Section V-B. However, the out-of-sample scenarios are different from the in-sample scenarios of the stochastic model. After generating 1000 in-sample scenarios, 10 most diverse and probable scenarios have been selected among them using an efficient scenario reduction tool, named SCENRED2, provided by the GAMS software [43]. The employed stochastic model is two-stage stochastic programming and it has been solved in unified form. Using decomposition techniques does not change the computation time of the stochastic model significantly. Considering the 10 selected scenarios, the compact formulation of the stochastic micro-CHP planning model can be presented as (A1) and (A2), subject to (7)-(31) for each scenario s':

$$C^{SA} = \min\left(C^{InvCHP} + C^{InvBoiler} + C^{InvTank} + \sum_{s' \in NS'} \bar{w}_{s'} C_{s'}^{operation}\right) \quad \text{(A1)}$$

$$C_{s'}^{operation} = \sum_{t \in T} \frac{1}{(1+I)^t} \left(C_{t,s'}^{OpeCHP} + C_{t,s'}^{OpeBoiler} + C_{t,s'}^{Ele} + C_{t,s'}^{MT} - C_{t,s'}^{Sell}\right) \quad \forall s' \quad \text{(A2)}$$

The objective of the above stochastic model (C^{SA}) consists of investment costs (C^{InvCHP}, $C^{InvBoiler}$, and $C^{InvTank}$), the net present value of operation costs ($C_{t,s'}^{OpeCHP}$, $C_{t,s'}^{OpeBoiler}$, $C_{t,s'}^{Ele}$, and $C_{t,s'}^{MT}$), and the net present value of the benefit obtained from selling electricity to the grid ($C_{t,s'}^{Sell}$). The operation cost of each in-sample scenario s' ($C_{s'}^{operation}$) is included in (A1) considering its normalized probability $\bar{w}_{s'}$, which is computed similarly to $\bar{w}_s$ given in (40). Constraints (14)-(31) should be considered for each in-sample scenario s' of the stochastic micro-CHP planning model.

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