Optimal Equilibrium Selection of Price-maker Agents in Performance-based Regulation Market

Mohammad Rayati, Aras Sheikhi, Ali Mohammad Ranjbar, and Wei Sun

Abstract—**This paper analyzes the oligopolistic equilibria of multiple price-maker agents in performance-based regulation (PBR) markets. In these markets, there are price-maker agents representing some frequency regulation (FR) providers and a number of independent price-taker FR providers. A model of equilibrium problem with equilibrium constraints (EPECs) is employed in this paper to study the equilibria of a PBR market in the presence of price-maker agents and price-taker FR providers. Due to the incorporation of the FR providers' dynamics, the proposed model is reformulated as a mixed-integer linear programming (MILP) problem over innovative mathematical techniques. An optimal equilibrium point is also selected for the market, where none of the agents is the unique deviator and the dynamic performance of power system is improved simultaneously. The effectiveness of the proposed optimal equilibrium point is evaluated by comparing the outputs with the conventional optimal dispatches of the FR providers.**

Index Terms—**Performance-based regulation (PBR) market, multi-agent system, equilibrium problem with equilibrium constraint (EPEC), frequency regulation (FR), mixed-integer linear programming (MILP).**

Nomenclature

A. Indices

ω	Index of scenarios
i	Index of price-maker agents
j, k	Indices of frequency regulation (FR) providers
t, τ	Indices of time-slots

B. Sets

Θ_i	Set of FR providers belonging to agent i
Ω	Set of scenarios
I	Set of price-maker agents
J	Set of price-maker FR providers
K	Set of price-taker FR providers
T	Set of time-slots

C. Variables

π_j^C	Capacity price of FR provider j
π_j^M	Mileage price of FR provider j
λ^C	Cleared capacity price
λ^M	Cleared mileage price
$\kappa_{j\omega}$	Performance score of FR provider j in scenario ω
$\eta_{j\omega}$	Constant of instructed automatic generation control (AGC) signal for FR provider j in scenario ω
$m_{j\omega}$	Actual mileage of FR provider j in scenario ω
r_j^M	Mileage allocation of FR provider j
r_j^C	Capacity allocation of FR provider j
$R_{j\omega}$	Reward of FR provider j in scenario ω
$s_{j\omega,t}$	Allocated AGC signal to FR provider j at time-slot t in scenario ω
$y_{j\omega,t}$	Response of FR provider j at time-slot t in scenario ω

D. Constants and Parameters

Δt_P	PBR market interval
Δt_A	Time resolution of AGC signal
σ_j	Mileage multiplier of FR provider j
M	A fixed large number
O_j^C	Capacity cost of FR provider j
O_j^M	Mileage cost of FR provider j
q	A binary variable
R^C	Regulation capacity requirement
R^M	Regulation mileage requirement
$s_{\omega,t}$	AGC signal at time-slot t in scenario ω
T_j	Time constant of FR provider j
U_j	Regulation capacity of FR provider j
v_ω	Probability of scenario ω

Manuscript received: August 22, 2019; revised: December 3, 2019; accepted: July 10, 2020. Date of CrossCheck: July 10, 2020. Date of online publication: November 20, 2020.

M. Rayati (corresponding author) and A. M. Ranjbar are with the Department of Electrical Engineering, Sharif University of Technology, Tehran, Iran (e-mail: mohammad.rayiati@gmail.com; amranjbar@sharif.edu).

A. Sheikhi is with Sharif Energy Research Institute (SERI), Sharif University of Technology, Tehran, Iran (e-mail: asheikhi@sharif.edu).

W. Sun is with the Department of Electrical Engineering and Computer Science, University of Central Florida, Orlando, FL 32816, USA (e-mail: sun@ucf.edu).

DOI: 10.35833/MPCE.2019.000559

I. Introduction

THE concept of smart grid is a paradigm in response to the sustainability challenges of energy production and consumption. A smart grid should be perfectly managed in the presence of fast integration of intermittent renewable energy sources (RESs) and inevitable fluctuations in load demands. The frequency declination, power system instability, and blackouts occurred due to the differences between energy production and consumption. In such cases, it is vital to balance the power flow and restore the frequency within the prescribed limitations at the earliest possible time [1]. In this regard, a number of systems, sub-systems, and markets are presented in smart grids to balance the differences of supply and demand.

In view of an independent system operator (ISO), the frequency regulation (FR) providers, e. g., energy storages and electric vehicle aggregators, participate in the automatic generation control (AGC) system to keep the demand and supply in balance. As a result, the frequency is maintained within a suitable margin. One of the most significant objectives of ISOs is the perfect design of the AGC system in order to optimize the operation of FR providers as well as meeting the electric grid performance criteria [2]. These objectives can be achieved by introducing a proper market mechanism for dispatching the FR providers in the operational time scales.

The AGC system is one of the effective tools for the ISOs specifically in high penetration of RESs to keep the frequency close to the nominal values, maintain tie-lines at the scheduled values, and optimize the allocation of AGC dispatch among FR providers [3]. In terms of fairness, ISOs should discriminate high- and low-quality FR providers. In other words, high-quality FR providers, who participate more effectively in the AGC system, shall be remunerated appropriately based on their provided services. To deal with this concern, the performance-based regulation (PBR) market has been established based on Federal Energy Regulatory Commission (FERC) Order 755 to determine the participation factors of FR providers in the AGC system [4].

Before FERC Order 755, the market mechanisms of remunerations for regulation services were based on reserved capacities of the FR providers, i.e., their head-rooms. However, as stated in the issued rule of October 20, 2011, the payment mechanisms based on unoccupied capacities are "unjust, unreasonable, and unduly discriminatory or preferential" [4]. To address this incompetency, the PBR market mechanism is introduced to manage the FR providers fairly, where the payments are based on the reserved capacities and the deployed mileages. Moreover, the mileage parts of FR providers' revenues are multiplied by the respected performance scores to reward the FR providers with better qualities in terms of tracking accuracy or delay of the AGC signal [4]. Despite all intuitive superiorities of the PBR markets compared with the reserve-based capacity markets, the specific following points can be indicated:

1) After the implementation of PBR markets, the FR providers are motivated to provide a better quality of services. However, the performances of FR providers depend on their dynamics, i. e., technological specifications. As long as the FR providers' revenues in the PBR markets are uncompetitive, they are not interested in participating in the market. Therefore, the price-maker agents are appearing to manage a number of FR providers to make this market feasible [5]. With the aggregation of price-maker agents, an oligopolistic equilibrium for the PBR markets is proposed in this paper.

2) By using the PBR market, there is a discrimination between the FR providers with different capacities and mileages/qualities. Due to their technical flexibilities/specifications, a number of FR providers can give more capacities with less mileages/qualities. In this paper, they are called fast-ramping FR providers. Note that the price-maker agents can manipulate their offers and cause an inefficient outcome. In this paper, an optimal equilibrium point for a PBR market is proposed, where none of the price-maker agents is the unique deviator and the dynamic performance of power system is enhanced simultaneously.

There are few studies about the efficiency of the PBR markets in the literature. However, these few studies can be categorized into three areas: ① analyzing the different ISO rules [6]-[10]; ② studying the FR providers' performances [11]-[15]; and ③ addressing the behaviors of price-taker and price-maker agents in the PBR markets [16]-[19].

Regarding to the first category, a number of ISOs in USA run the PBR market with different approaches in calculating the performance scores of FR providers and agents. Moreover, several ISOs determine a minimum threshold for participation of the FR providers and the agents in the PBR market. In [6], a study has been proposed on the performance score calculation methods of different ISOs. The effectiveness of the PBR market in Pennsylvania, New-Jersey, and Maryland (PJM) interconnection has been demonstrated in [7]. The advantages and disadvantages of performance score calculation methods of California ISO (CAISO) and Midwest ISO (MISO) have been mathematically explained and compared in [8]. The overview of PBR market implementation by MISO has also been explained in [9]. The MISO market rules have been enhanced in [10] to comply with FERC Order 755 by designing an optimal market clearing process and new measurement rules for performance accuracy.

With respect to the second category, one of the main objectives of an efficient PBR market mechanism is that the fast-ramping FR providers, e.g., energy storages, contribute a large portion of the required regulation services. As energy storages have better ramping characteristics compared with the traditional generation units, it has been recommended in [11] to find an optimal market mechanism to facilitate their participations in FR. A PBR market mechanism has been appropriately designed in [12], in which the fast-ramping energy storages have a higher priority to be selected in the market. A comparison on the policies of different implementations of the PBR markets has been conducted in [13] to analyze the impacts of policies on the participations of energy storages. Despite all advantages of energy storages, to address their high investment costs, another approach has been developed by obtaining the required ancillary services from the wind power plants. The dynamic of a wind power plant with the capability of providing the FR services has been simulated in [14], where the performance scores and revenues of a wind power plant in the PBR markets of CAISO and PJM are evaluated and compared. The performance scores of a wind farm providing the regulation services have also been analyzed in [15] from an experimental point of view.

In the third category, it is assumed that the fast-ramping FR providers become price-makers in the PBR markets. Note that the price-taker FR providers participate in the PBR markets by offering their marginal operation costs of capacity and mileage terms or optimizing their offers using stochastic/robust optimization methods. In practice, a PBR market runs with just few price-maker agents providing regulation services. In [16], an optimization model for an agent coordinating a number of energy storages has been proposed in a simultaneous energy and PBR market. In [17], an optimal bidding strategy of energy storages in the PBR market of PJM has been proposed by considering the impacts of energy storage cycle life on the total profit of price-taker agent. In [18], the strategic bidding of an agent consisting of a number of FR providers in the PBR market of CAISO has been determined using the formulation of mathematical programming with equilibrium constraints (MPECs) and nonlinear programming technique. Finally, the MPEC problem of a price-maker agent in the PBR market of CAISO has been transformed into a mixed-integer linear programming (MILP) problem in [19] considering the details of PBR market, e.g., FR providers' dynamics and performance score calculation method of CAISO.

Our study is categorized in the third category. Due to the imperfect competition and information asymmetries, there are gaming opportunities for the FR providers and price-maker agents to increase their profits by the manipulation of capacities and mileage offering prices in the PBR markets. Up to our knowledge, none of the previous works studies the equilibrium of the PBR market with more than one price-maker agent. The oligopolistic equilibrium in a PBR market is analyzed using the model of equilibrium problem with equilibrium constraints (EPECs), which is widely studied in the literature of electricity markets. Due to some restrictions of FR providers' dynamics, new challenges should be addressed in solving the proposed EPEC model.

The major contributions of this paper are as follows.

1) A stochastic real-time PBR market including a number of price-maker agents and price-taker FR providers is modeled based on EPEC formulation.

2) The details of the PBR market such as performance score calculation method and scenarios of AGC signal in real ime are considered in the formulations. Compared with [17], where the bidding strategy of a price-taker energy storage has been found considering the details of PBR markets such as performance score calculation methods and scenarios of AGC signal in real time, the EPEC formulation in this paper is developed for more than one price-maker agent. Moreover, the dynamics of FR providers in the AGC system are modeled.

3) The proposed EPEC formulation for finding the equilibria has complex nonlinear terms. In this paper, by providing innovative mathematical techniques, the proposed nonlinear problem is reformulated as an MILP formulation.

4) Compared with [19], an additional theorem is given in this paper to linearize the proposed EPEC problem. As a result, the linearization technique of this paper has differences with [19] and leads to less variables/constraints for writing the EPEC problem. It is worth mentioning that the model of [19] is an MPEC problem for a single price-maker agent and its linearization technique cannot be easily extended to multiple price-maker agents.

5) In the optimal equilibrium of the PBR market, in addition to the dynamic performance enhancement of the power system, none of the price-maker agents has any incentive for deviation.

The remainder of this paper is organized as follows. A PBR market model is mathematically formulated in Section II. The EPEC model is presented in Section III for finding the equilibria of the PBR market. In addition, to solve the EPEC model by commercial solvers, the procedure of converting the proposed EPEC model into an MILP problem is elaborated in Section III. The proposed optimal equilibrium is also introduced in Section IV. The performance of the proposed model is evaluated by simulating on a test system in Section V. Finally, concluding remarks and future works are given in Section VI.

II. Mathematical Formulation of PBR Market

Before formulating the problem, it is noteworthy that the PBR market is elaborated. To ensure the quality and stability of energy supply, the ISO purchases an ancillary service called FR. The generation units, consumers, or energy storages that are willing to provide such service submit their capacity and mileage offering prices. The PBR market time interval Δt_P is equal to the real-time market time interval, e.g., 900 s (15 min). The regulation capacity is defined as "an unloaded capacity synchronizing with the system and ready to serve an additional demand" [4]. Moreover, "the absolute amount of injected or withdrawn energy by an FR provider in the AGC system" is called regulation mileage [4].

In the real-time operation, the obtained regulation capacity and mileage schedules are the inputs of the AGC system. The ISO allocates the AGC dispatch based on the results of the PBR market in the time resolution of the AGC system, i.e., Δt_A= 4 s. Finally, the FR providers receive the rewards based on their dynamics, their performances, and the realized AGC scenario.

A. Formulation of PBR Market

In a PBR market, there are a number of price-maker agents and price-taker FR providers. It is also assumed that a price-maker agent can monitor and manage the operation of a number of FR providers. An agent refers to a price-maker entity who bids for a number of FR providers.

The PBR market problem is formulated in (1)-(5).

$$\min \sum_{j \in J \cup K} \left(\pi_j^C r_j^C + \pi_j^M r_j^M \right) \tag{1}$$

s.t.

$$\sum_{j \in J \cup K} r_j^C \geq R^C \quad (\lambda^C) \tag{2}$$

$$\sum_{j \in J \cup K} r_j^M \geq R^M \quad (\lambda^M) \tag{3}$$

$$0 \leq r_j^C \leq U_j \quad \forall j \in J \cup K; (\mu_j^{C1}, \mu_j^{C2}) \tag{4}$$

$$r_j^C \leq r_j^M \leq \sigma_j r_j^C \quad \forall j \in J \cup K; (\mu_j^{M1}, \mu_j^{M2}) \tag{5}$$

Formula (1) represents the objective function of the PBR market problem including the sum of the FR providers' bidding offers. Constraints (2) and (3) ensure that the market regulation capacity and mileage requirements are provided. In addition, constraint (4) enforces the regulation capacity limitations of the FR providers. Finally, the limitations of the regulation mileage are enforced for all FR providers through (5). It is worth mentioning that λ^C and λ^M are dual variables of constraints (2) and (3), respectively. Moreover, μ_j^{C1}, μ_j^{C2}, μ_j^{M1}, and μ_j^{M2} are dual variables of constraints (4) and (5). Dual variables μ_j^{C1} and μ_j^{M1} correspond to the minimum constraints of r_j^C and r_j^M. Concurrently, dual variables μ_j^{C2} and μ_j^{M2} correspond to the maximum constraints of r_j^C and r_j^M.

Remark 1: the regulation mileage of an FR provider in (5) is limited by the multiplication of the cleared regulation capacity and a mileage multiplier, i.e., σ_j, which is calculated based on the previous historical performance of the FR provider. Parameter σ_j is the ratio of the total regulation mileage, which has been actually provided by the FR provider, and the total procured regulation capacity from the FR provider in the same operation time interval over the previous week/month. By using this parameter, the FR providers' mileage is not more than its practical capability to follow the AGC signal.

Remark 2: the other point that should be noted is the dual variables of constraints, which are presented in (1)-(5), e.g., λ^C and λ^M. They are shadow prices of these constraints and indicate the opportunity costs from the provision of the regulation capacity and mileage. These costs are reflected on the submitted bids of price-maker agents and price-taker FR providers, i.e., π_j^C and π_j^M. Thus, these shadow prices (λ^C and λ^M) are employed for rewarding the FR providers.

Remark 3: the ISO and agents do not have the detailed private information of the FR providers. Due to the asymmetry of the information, the agents act strategically and do not reveal their private information in the PBR market to gain more profits. In this paper, the equilibrium of these price-maker agents in the PBR market is analyzed.

B. Formulation of Allocation of Instructed AGC Signal

After clearing the PBR market, the instructed AGC signal is allocated among the FR providers based on their cleared mileages and capacities as formulated in (6).

$$s_{j\omega,t} = \min\left\{ \frac{r_j^M}{\sum\limits_{k \in J \bigcup K} r_k^M} s_{\omega,t}, r_j^C \right\} \quad \forall j \in J \bigcup K, \forall \omega \in \Omega, \forall t \in T \tag{6}$$

The instructed AGC signal to an FR provider for a possible scenario of AGC signal, i.e., $\forall \omega \in \Omega$, is based on the cleared mileage allocation as formulated in the first term of (6). Moreover, the allocated AGC signal is limited to the cleared capacity allocation as shown in the second term of (6).

Assumption 1: for all FR providers, it is presumed that $\sigma_j \leq R^M / R^C$. Note that this assumption is practically correct as pointed out in [19].

If Assumption 1 holds, (6) can be replaced by (7) for allocating the instructed AGC signal among the FR providers. The proof can be found in [19].

$$s_{j\omega,t} = r_j^M s_{\omega,t} / R^M \tag{7}$$

C. Formulation of System Dynamic of FR Provider

After the allocation of the instructed AGC signal, the FR providers respond to it based on their dynamics. An FR provider can be a governor-turbine, which is often modeled in the simplest way as a first-order dynamic system that has a time constant representing the dynamic of its governor and turbine. Other technologies can also be considered depending on the generation technology. Therefore, other dynamic system models can be employed [20]. The dynamic of an FR provider with a simple structure is formulated in (8). This dynamic system has just one time constant. It is inspired from the model presented in [19].

$$T_j \frac{\partial y_{j\omega,t}}{\partial t} = s_{j\omega,t} - y_{j\omega,t} \tag{8}$$

By applying the inverse Laplace transform to (8), the actual response of an FR provider is formulated as (9).

$$y_{j\omega,t} = \sum_{\tau=1}^{t} \left(s_{j\omega,\tau} - s_{j\omega,\tau-1} \right) \left(1 - \mathrm{e}^{\frac{-(t-\tau+1)\Delta t_A}{T_j}} \right) u(t-\tau+1) + s_{j\omega,0} \mathrm{e}^{-\frac{t\Delta t_A}{T_j}} \tag{9}$$

where $u(t) = 1, \forall t \geq 0$ and $u(t) = 0, \forall t < 0$.

The actual mileage of an FR provider in a scenario with the instructed AGC signal for the PBR market interval, e.g., 15 min, is formulated as (10).

$$m_{j\omega} = \sum_{t=1}^{\Delta t_P / \Delta t_A} \left| y_{j\omega,t} - y_{j\omega,t-1} \right| \tag{10}$$

The actual mileage in (10) is the sum of the upward and downward absolute movements of the FR providers to follow the instructed AGC signal.

D. Formulation of Payment to FR Provider

The FR providers are rewarded based on (11).

$$R_{j\omega} = \lambda^C r_j^C + \lambda^M m_{j\omega} \kappa_{j\omega} \tag{11}$$

Remark 4: parameter $\kappa_{j\omega}$ is used to capture the performance score of FR providers. The mileage should be adjusted to reflect the actual contribution performance of the FR providers. The performance score shows how well an FR provider follows the dispatching commands within the PBR market time interval. In CAISO, it is calculated by (12).

$$\kappa_{j\omega} = 1 - \frac{\sum\limits_{t=1}^{\Delta t_P / \Delta t_A} \left| s_{j\omega,t} - y_{j\omega,t} \right|}{\sum\limits_{t=1}^{\Delta t_P / \Delta t_A} \left| s_{j\omega,t} \right|} \tag{12}$$

Theorem 1: if Assumption 1 holds, (11) can be rewritten as (13).

$$R_{j\omega} = \lambda^C r_j^C + \lambda^M \eta_{j\omega} r_j^M \tag{13}$$

The proof of Theorem 1 is given in Appendix A.

E. Problem Definition

As explained, it is pivotal to consider a remuneration mechanism for the FR providers. As described in the introduction, the PBR market covers all concerns related to the remuneration of the FR providers.

In a traditional power system, the ISO has controlled over the operation of all system components. Hence, it can manage the components using control and optimization techniques. However, in the presence of price-maker participants, an agent acts autonomously considering its knowledge about the overall operation of the network as well as the behaviors of other agents over time. Combining this information along with the expectation of an agent about the behaviors of others agents, an agent makes decisions in real time to maximize its own benefits. The decisions of the agent, in turn, affect the evolution of the network over time and determine its overall performance. Therefore, it is important how price-maker agents and price-taker FR providers interact. In the next section, the outputs of this interaction are studied.

III. Equilibria of PBR Market

The EPEC mathematical model of the game among the price-maker agents in the PBR market is written in this section. To this end, the optimization problem of a price-maker agent is given. Then, the MPEC reformulation of problems of the price-maker agents is presented and the EPEC formulation for finding the equilibria of the PBR markets is proposed.

A. Optimization Problem of Price-maker Agent

A price-maker agent solves the optimization problem with (1)-(5), (14)-(16) to find its optimal offering prices in the PBR market. An agent includes a certain number of the FR providers, i.e., $j \in \Theta_i$.

$$\max \sum_{j \in \Theta_i} \sum_{\omega \in \Omega} v_\omega \left(R_{j\omega} - O_j^C r_j^C - O_j^M m_{j\omega} \right) \tag{14}$$

s.t.

$$0 \le \pi_j^C \le \bar{\pi}^C \quad \forall j \in \Theta_i \tag{15}$$

$$0 \le \pi_j^M \le \bar{\pi}^M \quad \forall j \in \Theta_i \tag{16}$$

The objective function (14) corresponds to the expected profit of agent i. Note that the profit of the agent equals to the difference between the rewards to its FR providers and their operation costs. Constraints (15) and (16) show that the offering capacity and mileage prices are positive and limited in the acceptable ranges, i. e., $[0, \bar{\pi}^C]$ and $[0, \bar{\pi}^M]$. The PBR market problem is also considered in the modeling using (1)-(5) as a sub-level of the proposed optimization problem. Note that the FR providers receive their rewards based on the cleared dual variables of the PBR market, i.e., λ^C and λ^M.

B. MPEC Reformulation

To solve the bi-level optimization problem in (14)-(16) with (1)-(5), sub-level problem in (1)-(5) should be replaced with the equivalent conditions to obtain the MPEC reformulation. The PBR market problem, stated in (1)-(5), is convex and satisfies Slater's constraint qualification. Hence, the strong duality holds for this problem and the first-order Karush-Kuhn-Tucker (KKT) conditions are both necessary and sufficient for optimality [21], which are presented in (17)-(21).

$$\sum_{j \in J \cup K} \left(\pi_j^C r_j^C + \pi_j^M r_j^M + \mu_j^{C2} U_j \right) - \lambda^C R^C - \lambda^M R^M = 0 \quad (\beta_i^R) \tag{17}$$

$$\pi_j^C - \lambda^C - \mu_j^{C1} + \mu_j^{C2} + \mu_j^{M1} - \sigma_j \mu_j^{M2} = 0 \quad \forall j \in J \cup K; (\beta_{ij}^C) \tag{18}$$

$$\pi_j^M - \lambda^M - \mu_j^{M1} + \mu_j^{M2} = 0 \quad \forall j \in J \cup K; (\beta_{ij}^M) \tag{19}$$

$$\lambda^C, \lambda^M, \mu_j^{C1}, \mu_j^{C2}, \mu_j^{M1}, \mu_j^{M2} \ge 0 \quad \forall j \in J; (\beta_i^{\lambda C}, \beta_i^{\lambda M}, \underline{\beta}_{ij}^{\mu C}, \bar{\beta}_{ij}^{\mu C}, \underline{\beta}_{ij}^{\mu M}, \bar{\beta}_{ij}^{\mu M}) \tag{20}$$

$$\begin{cases} \sum\limits_{j \in J \cup K} r_j^C \ge R^C \quad (\underline{\beta}_i^C) \\ \sum\limits_{j \in J \cup K} r_j^M \ge R^M \quad (\underline{\beta}_i^M) \\ 0 \le r_j^C \le U_j \quad \forall j \in J \cup K; (\underline{\beta}_{ij}^{rC}, \bar{\beta}_{ij}^{rC}) \\ r_j^C \le r_j^M \le \sigma_j r_j^C \quad \forall j \in J \cup K; (\underline{\beta}_{ij}^{rM}, \bar{\beta}_{ij}^{rM}) \end{cases} \tag{21}$$

These constraints replace (1)-(5) in the optimization problem of an agent. In (17)-(21), the corresponding dual variables of the constraints are presented. In (17), the strong duality condition of PBR market is presented instead of complementarity constraints, as it simplifies the derivation of EPEC formulation in the next subsection.

C. EPEC Formulation

The EPEC formulation is presented in (17)-(43). The EPEC is a joint solution of all agents whose equilibria are analyzed through the solutions associated with the KKT (strong stationary) conditions of (14)-(16) and (1)-(5) for all agents [22].

$$\sum_{\omega \in \Omega} v_\omega \left(O_j^M \frac{\partial m_{j\omega}}{\partial r_j^M} - \frac{\partial R_{j\omega}}{\partial r_j^M} \right) - \underline{\beta}_i^M - \underline{\beta}_{ij}^{rM} + \bar{\beta}_{ij}^{rM} + \beta_i^R \pi_j^M = 0 \quad \forall i \in I, \forall j \in \Theta_i \tag{22}$$

$$-\underline{\beta}_i^M - \underline{\beta}_{ij}^{rM} + \bar{\beta}_{ij}^{rM} + \beta_i^R \pi_j^M = 0 \quad \forall i \in I, \forall j \notin \Theta_i \tag{23}$$

$$O_j^C - \lambda^C - \underline{\beta}_i^C - \underline{\beta}_{ij}^{rC} + \bar{\beta}_{ij}^{rC} + \underline{\beta}_{ij}^{rM} - \sigma_j \bar{\beta}_{ij}^{rM} + \beta_i^R \pi_j^C = 0 \quad \forall i \in I, \forall j \in \Theta_i \tag{24}$$

$$-\underline{\beta}_i^C - \underline{\beta}_{ij}^{rC} + \bar{\beta}_{ij}^{rC} + \underline{\beta}_{ij}^{rM} - \sigma_j \bar{\beta}_{ij}^{rM} + \beta_i^R \pi_j^C = 0 \quad \forall i \in I, \forall j \notin \Theta_i \tag{25}$$

$$\beta_{ij}^M + \beta_i^R r_j^M - \underline{\beta}_j^{\pi M} + \bar{\beta}_j^{\pi M} = 0 \quad \forall i \in I, \forall j \in \Theta_i \tag{26}$$

$$\beta_{ij}^C + \beta_i^R r_j^C - \underline{\beta}_j^{\pi C} + \bar{\beta}_j^{\pi C} = 0 \quad \forall i \in I, \forall j \in \Theta_i \tag{27}$$

$$-\sum_{\omega \in \Omega} \sum_{j \in \Theta_i} v_\omega m_{j\omega} \kappa_{j\omega} - \sum_{j \in J \cup K} \beta_{ij}^M - \beta_i^R R^M - \beta_i^{\lambda M} = 0 \quad \forall i \in I \tag{28}$$

$$-\sum_{j \in \Theta_i} r_j^C - \sum_{j \in J \cup K} \beta_{ij}^C - \beta_i^R R^C - \beta_i^{\lambda C} = 0 \quad \forall i \in I \tag{29}$$

$$\beta_{ij}^C - \beta_{ij}^M - \underline{\beta}_{ij}^{\mu M} = 0 \quad \forall i \in I, \forall j \in J \cup K \tag{30}$$

$$-\sigma_j \beta_{ij}^C + \beta_{ij}^M - \bar{\beta}_{ij}^{\mu M} = 0 \quad \forall i \in I, \forall j \in J \cup K \tag{31}$$

$$-\beta_{ij}^C - \underline{\beta}_{ij}^{\mu C} = 0 \quad \forall i \in I, \forall j \in J \cup K \tag{32}$$

$$\beta_{ij}^C + \beta_i^R U_j - \bar{\beta}_{ij}^{\mu C} = 0 \quad \forall i \in I, \forall j \in J \cup K \tag{33}$$

$$\begin{cases} \lambda^C \beta_i^{\lambda C} = 0 \\ \beta_i^{\lambda C} \ge 0 \end{cases} \quad \forall i \in I \tag{34}$$

$$\begin{cases} \lambda^M \beta_i^{\lambda M} = 0 \\ \beta_i^{\lambda M} \ge 0 \end{cases} \quad \forall i \in I \tag{35}$$

$$\begin{cases}\mu_j^{C1}\underline{\beta}_{ij}^{\mu C}=0\\ \underline{\beta}_{ij}^{\mu C}\geq 0\end{cases}\quad \forall i\in I,\forall j\in J\bigcup K \tag{36}$$

$$\begin{cases}\mu_j^{C2}\bar{\beta}_{ij}^{\mu C}=0\\ \bar{\beta}_{ij}^{\mu C}\geq 0\end{cases}\quad \forall i\in I,\forall j\in J\bigcup K \tag{37}$$

$$\begin{cases}\mu_j^{M1}\underline{\beta}_{ij}^{\mu M}=0\\ \underline{\beta}_{ij}^{\mu M}\geq 0\end{cases}\quad \forall i\in I,\forall j\in J\bigcup K \tag{38}$$

$$\begin{cases}\mu_j^{M2}\bar{\beta}_{ij}^{\mu M}=0\\ \bar{\beta}_{ij}^{\mu M}\geq 0\end{cases}\quad \forall i\in I,\forall j\in J\bigcup K \tag{39}$$

$$\begin{cases}\pi_j^{C}\underline{\beta}_{j}^{\pi C}=0\\ \pi_j^{C}\geq 0\\ \underline{\beta}_{j}^{\pi C}\geq 0\end{cases}\quad \forall j\in \Theta_i \tag{40}$$

$$\begin{cases}(-\pi_j^{C}+\bar{\pi}^{C})\bar{\beta}_{j}^{\pi C}=0\\ \bar{\pi}^{C}\geq \pi_j^{C}\\ \bar{\beta}_{j}^{\pi C}\geq 0\end{cases}\quad \forall j\in \Theta_i \tag{41}$$

$$\begin{cases}\pi_j^{M}\underline{\beta}_{j}^{\pi M}=0\\ \pi_j^{M}\geq 0\\ \underline{\beta}_{j}^{\pi M}\geq 0\end{cases}\quad \forall j\in \Theta_i \tag{42}$$

$$\begin{cases}(-\pi_j^{M}+\bar{\pi}^{M})\bar{\beta}_{j}^{\pi M}=0\\ \bar{\pi}^{M}\geq \pi_j^{M}\\ \bar{\beta}_{j}^{\pi M}\geq 0\end{cases}\quad \forall j\in \Theta_i \tag{43}$$

Note that the proposed EPEC is nonlinear and highly nonconvex. Hence, the proposed EPEC problem should be reformulated as an MILP problem.

IV. Optimal Equilibrium Selection

A model of the game among price-maker agents in the PBR market is presented in Section III. Although the gradient-based methods can be employed to analyze the mentioned game equilibria, the proposed formulation has the following merits compared with the gradient-based methods: ① the proposed formulation can find the equilibria in one shot, if it is reformulated as an MILP problem; and ② the convergence of the gradient-based methods cannot be guaranteed.

As the derived EPEC formulation is nonlinear and cannot be solved by commercial solvers, the conversion procedure of the modeling into an MILP problem is presented in this section. Throughout five mathematical steps, the problem is reformulated as an MILP problem. However, an appropriate objective function is introduced firstly to select an optimal equilibrium among existing equilibria.

A. Optimal Equilibrium

Solving the EPEC problem leads to a number of equilibria. In this paper, an optimal equilibrium is selected, which can be calculated by solving (44).

$$\begin{cases}\max \sum_{\omega\in\Omega}\sum_{j\in J\bigcup K} v_\omega \eta_{j\omega} r_j^M\\ \text{s.t. } (22)\text{-}(43)\end{cases} \tag{44}$$

The objective function in (44) includes the expected sum of actual mileage multiplied by performance score. Note that based on Theorem 1, $\eta_{j\omega}r_j^M$ is equal to $\kappa_{j\omega}m_{j\omega}$. With this optimal equilibrium, the high-quality FR providers or fast-ramping FR providers have priority to be selected compared with the low-quality ones.

B. MILP Problem Formulation

The derived EPEC model is nonlinear. To convert it into an MILP problem, the following steps need to be taken. After this conversion, the problem can be solved by interior point method (IPM) solvers.

1) The first source of nonlinearities is the multiplication of π_j^M and π_j^C for all price-taker FR providers by β_i^R in (23) and (25), respectively. The optimization problem of a price-maker agent is given in (1)-(5) and (14)-(16). By eliminating (1)-(5) from the constraints and fixing λ^C and λ^M to the predicted values, (14) - (16) show the stochastic optimization problem of a price-taker FR provider. Then, the strategy of price-taker FR providers would be the respected marginal operation costs. As a result, variables π_j^M and π_j^C of price-taker FR providers should be replaced by the respected marginal operation costs.

2) Presence of actual mileage in the formulation. Herein, the payoffs of FR providers are calculated based on the actual mileage and performance scores, i.e., $m_{j\omega}$ and $\kappa_{j\omega}$. By using Theorem 1, $m_{j\omega}$ can be replaced by $\eta_{j\omega}r_j^M/\kappa_{j\omega}$.

3) Strong duality condition replacement. Another source of nonlinearity is (17), which represents the strong duality condition. It can be replaced by its equivalent complementarity constraints in (45) - (50). However, the complementarity constraints are other sources of nonlinearity which are linearized in the next step.

$$\left(\sum_{j\in J\bigcup K} r_j^C - R^C\right)\lambda^C = 0 \tag{45}$$

$$\left(\sum_{j\in J\bigcup K} r_j^M - R^M\right)\lambda^M = 0 \tag{46}$$

$$r_j^C\mu_j^{C1}=0\quad \forall j\in J\bigcup K \tag{47}$$

$$(U_j - r_j^C)\mu_j^{C2}=0\quad \forall j\in J\bigcup K \tag{48}$$

$$(\sigma_j r_j^C - r_j^M)\mu_j^{M2}=0\quad \forall j\in J\bigcup K \tag{49}$$

$$(r_j^M - r_j^C)\mu_j^{M1}=0\quad \forall j\in J\bigcup K \tag{50}$$

It is worth remarking that this conversion seems to go against the statement made before about the computational advantages of using strong duality constraints. However, these advantages are obtained when the EPEC formulation is derived. Note that if the complementarity constraints are used from the first, the EPEC formulation would have been much more complicated.

4) Implementation of Big-M method. The complementarity constraints such as (45) - (50) are linearized using the equivalent constraints presented in (51).

$$\left(\beta, P \geq 0, \beta \leq qM, P \leq (1-q)M\right) \tag{51}$$

5) Parameterization. The last source of nonlinearities exists due to the multiplication of π_j^M and π_j^C for all FR providers belonging to price-maker agents by dual variable β_i^R. By parameterizing dual variable β_i^R, the problem becomes linear.

V. Simulation Result and Discussion

A. Basic Data

In this subsection, the performance of proposed method is evaluated by presenting a case study including 19 FR providers and 2 price-maker agents. The detailed information of agents and FR providers are presented in Table I. As observed, the first agent includes more fast-ramping FR providers with fewer total capacities compared with the second agent.

TABLE I
DATA OF AGENTS AND FR PROVIDER IN CASE STUDY

Agent	Total capacity (MW)	j	O_j^C (\$/MW)	O_j^M (\$/MW)	U_j (MW)	σ_j	T_j (s)
Agent 1	55.0	1	8	2	7.50	4	15.5
		2	8	2	12.50	4	9.5
		3	8	2	15.00	2	12.0
		4	5	2	12.50	4	10.0
		5	10	2	7.50	2	13.0
Agent 2	80.0	6	8	2	12.50	5	36.3
		7	4	2	20.00	3	33.6
		8	6	2	15.00	3	42.3
		9	10	2	20.00	3	57.0
		10	11	2	12.50	5	30.6
Price-taker FR provider	84.5	11	10	4	10.00	3	10.5
		12	10	4	10.00	4	10.7
		13	11	4	11.25	3	10.1
		14	11	4	12.50	4	15.0
		15	12	2	17.50	5	14.3
		16	12	2	2.50	2	9.0
		17	15	1	7.50	3	7.0
		18	9	6	8.25	4	18.0
		19	9	6	5.00	4	18.9

Herein, the proposed game dispatch is compared with a benchmark case, which is the optimal dispatch of the FR providers and agents. In the optimal dispatch, all the agents and FR providers are considered as price-takers and the dispatch is determined by (1)-(5) when variables π_j^M and π_j^C of all individuals and FR providers belonging to the agents are replaced by the respected marginal operation costs. This benchmark case has also been presented in [12].

In Figs. 1-5, five scenarios of the instructed AGC signal are illustrated with similar probability values. These data are obtained from CAISO database. For these scenarios, the capacity and regulation mileage requirements are 100 MW and 350 MW, respectively. The actual response of the FR providers in both cases of optimal dispatch and game dispatch are also illustrated in Figs. 1-5. Obviously, the performance of the responses of the FR providers is enhanced in the game dispatch compared with the optimal dispatch for all scenarios of AGC signals. It is due to the fact that the effective mileages of the FR providers are maximized in the game dispatch.

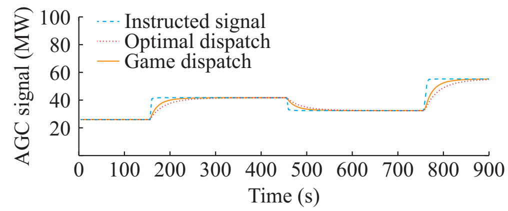

Fig. 1. Instructed AGC signal and response in the first scenario.

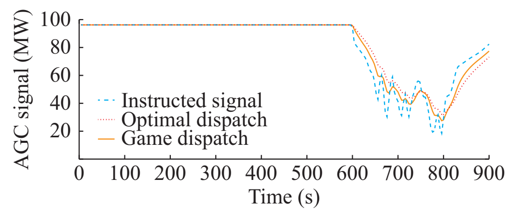

Fig. 2. Instructed AGC signal and response in the second scenario.

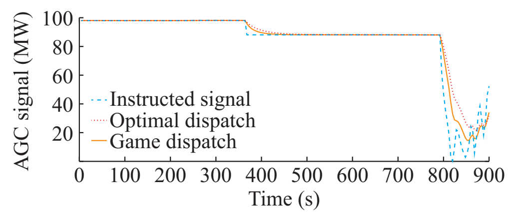

Fig. 3. Instructed AGC signal and response in the third scenario.

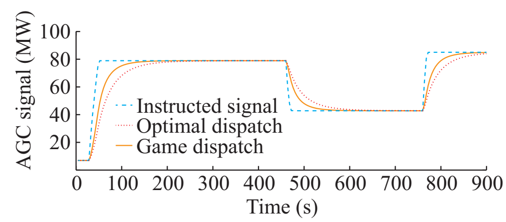

Fig. 4. Instructed AGC signal and response in the fourth scenario.

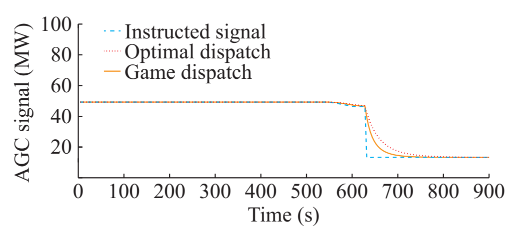

Fig. 5. Instructed AGC signal and response in the fifth scenario.

B. Total Cost and Accuracy of AGC System

In this subsection, the optimal dispatch and the game dispatch are compared in respect to two criteria: the social welfare and the dynamic performance of the power system. As observed in Fig. 6, the total cost and market clearing prices increase in the game dispatch. Note that the oligopolistic behaviors of agents cause this inefficiency of social welfare indices. However, as observed in Fig. 7, the dynamic performance of power system is improved in regard to the accuracy of AGC signal tracking in all scenarios.

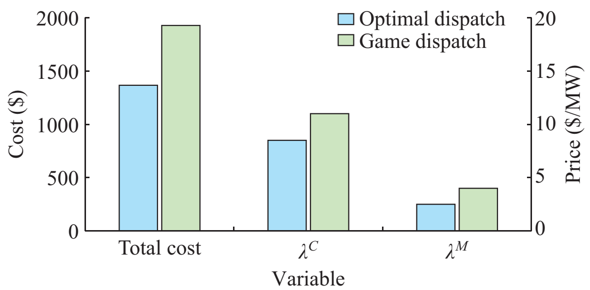

Fig. 6. Impacts of game dispatch on social welfare indices.

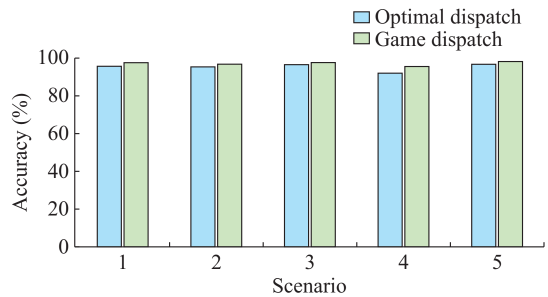

Fig. 7. AGC tracking accuracy of game dispatch and optimal dispatch.

It is worth mentioning that the accuracy of AGC signal is just slightly improved with the proposed game dispatch in Fig. 7, whereas the total cost is much higher in the proposed game dispatch than the optimal dispatch in Fig. 6. Therefore, the effectiveness and necessity of the proposed game dispatch seem unjustified. However, the optimal dispatch cannot be proposed for the price-maker agents as the main aim of this study is a strategy-proof dispatch for the FR providers belonging to the price-maker agents. In addition, the increment of the total cost for the game dispatch is trivial since the oligopolistic behaviors of the agents cause this inefficiency. To overcome this challenge and reduce the total cost (improve social welfare indices in general), a market mechanism for the price-maker agents should be designed.

C. Dispatch and Profit of FR Provider

The profits of price-maker agents and price-taker FR providers in the optimal dispatch and the game dispatch are shown and compared in Fig. 8. The interesting observation is that the price-maker agent with less qualities of FR providers (agent 2) is worse off in the game dispatch. It means that in the equilibrium point, the price-maker agent with higher quality of FR providers (agent 1) gains more payoffs.

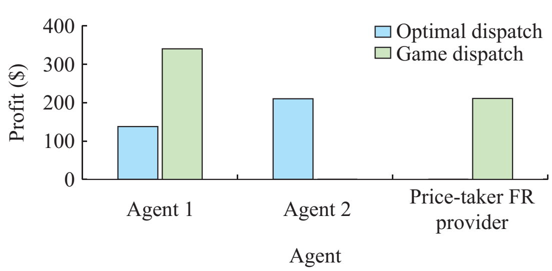

Fig. 8. Profits of price-maker agents and price-taker FR providers.

VI. Conclusion

In this paper, a method based on the EPEC model is proposed for evaluating the effectiveness of PBR market in the presence of price-maker agents consisting of a number of FR providers. An optimal equilibrium point is presented where the agents do not have any incentive for deviations. Moreover, the accuracy of actual time response of FR providers is enhanced in the proposed equilibrium point. By a theorem about specifications of the PBR market and throughout five steps, the proposed nonlinear model is reformulated as an MILP problem. Therefore, it can be solved in one shot. The effectiveness of proposed model is evaluated in the case study.

Due to the oligopolistic behaviors of price-maker agents, the profit of the agent that includes more fast-ramping FR providers has been more than doubled due to the fair remuneration mechanism. In future works, the mechanism of PBR markets can be modified so that the impacts of price-maker agents on the social welfare are mitigated. Moreover, a decentralized algorithm can be considered for price-maker agents to reach the equilibrium in an environment with the asymmetry of the information.

Appendix A

Using (7), (9) and (12), (A1) and (A2) are concluded.

$$\kappa_{j\omega}=1-\frac{\dfrac{r_j^M}{R^M}\sum_{t\in T}\left|s_{\omega,t}-\tilde{y}_{j\omega,t}\right|}{\dfrac{r_j^M}{R^M}\sum_{t\in T}\left|s_{\omega,t}\right|} \tag{A1}$$

$$\tilde{y}_{j\omega,t}=\sum_{\tau=1}^{t}\left(s_{\omega,\tau}-s_{\omega,\tau-1}\right)\left(1-e^{-\frac{(t-\tau+1)\Delta t_A}{T_j}}\right)u(t-\tau+1)+s_{\omega,0}e^{\frac{-t\Delta t_A}{T_j}} \tag{A2}$$

Moreover, using the definition of mileage in (10), (A3) is concluded.

$$m_{j\omega}=\frac{r_j^M}{R^M}\sum_{t=1}^{\Delta t_P/\Delta t_A}\left|\tilde{y}_{j\omega,t}-\tilde{y}_{j\omega,t-1}\right| \tag{A3}$$

Thus, using (A2) and (A3), (A4) is concluded.

$$\kappa_{j\omega}m_{j\omega}=\eta_{j\omega}r_j^M \tag{A4}$$

References

[1] A. Wood and B. Wollenberg, *Power Generation, Operation, and Control*. New York: John Wiley & Sons, 2012.

[2] T. Basso, "IEEE 1547 and 2030 standards for distributed energy resources interconnection and interoperability with the electricity grid," NREL, Golden, USA, Tech. Rep., 2014.

[3] H. Bevrani, *Robust Power System Frequency Control*. Berlin: Springer, 2014.

[4] Federal Energy Regulatory Commission. (2011, Oct.). Order 755: frequency regulation compensation in the organized wholesale power markets. [Online]. Available: https://www. jonesday. com/en/insights/2011/10/ferc-issues-order-no-755-frequency-regulation-compensation-in-the-organized-wholesale-power-markets

[5] M. Balandat, F. Oldewurtel, M. Chen *et al.*, "Contract design for frequency regulation by aggregations of commercial buildings," in *Proceedings of 2014 52nd Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, Monticello, USA, Feb. 2014, pp. 38-45.

[6] A. Papalexopoulos and P. Andrianesis, "Performance-based pricing of frequency regulation in electricity markets," *IEEE Transactions on Power Systems*, vol. 29, no. 1, pp. 441-449, Jan. 2014.
[7] Y. Xiao, Q. Su, F. Bresler *et al*., "Performance-based regulation model in PJM wholesale markets," in *Proceedings of IEEE PES General Meeting*, National Harbor, USA, Jul. 2014, pp. 1-9.
[8] A. Sadeghi and H. Mohsenian-Rad, "Performance accuracy scores in CAISO and MISO regulation markets: a comparison based on real data and mathematical analysis," *IEEE Transactions on Power Systems*, vol. 33, no. 3, pp. 3196-3198, May 2018.
[9] R. Miller, B. Venkatesh, and D. Cheng, "Overview of FERC Order 755 and proposed MISO implementation," in *Proceedings of IEEE PES General Meeting*, Vancouver, Canada, Nov. 2013, pp. 1-8.
[10] Y. Chen, R. Leonard, M. Keyser *et al*., "Development of performance-based two-part regulating reserve compensation on MISO energy and ancillary service market," *IEEE Transactions on Power Systems*, vol. 30, no. 1, pp. 142-155, Jan. 2015.
[11] M. Kintner-Meyer, "Regulatory policy and markets for energy storage in North America," *Proceedings of the IEEE*, vol. 102, no. 7, pp. 1065-1072, Jul. 2014.
[12] Z. Wang, J. Zhong, and J. Li, "Design of performance-based frequency regulation market and its implementations in real-time operation," *International Journal of Electrical Power & Energy Systems*, vol. 87, pp. 187-197, May 2017.
[13] B. Xu, Y. Dvorkin, D. Kirschen *et al*., "A comparison of policies on the participation of storage in US frequency regulation markets," in *Proceedings of 2016 IEEE PES General Meeting*, Boston, USA, Jul. 2016, pp. 1-8.
[14] J. Aho, L. Pao, P. Fleming *et al*. (2015, Feb.). Controlling wind turbines for secondary frequency regulation: an analysis of AGC capabilities under new performance based compensation policy. [Online]. Available: https://www.osti.gov/biblio/1172284
[15] E. Rebello, D. Watson, and M. Rodgers, "Performance analysis of a 10 MW wind farm in providing secondary frequency regulation: experimental aspects," *IEEE Transactions on Power Systems*, vol. 34, no. 4, pp. 3090-3097, Jul. 2019.
[16] Y. Yao, P. Zhang, and S. Chen. (2019, Jan.). Aggregating large-scale generalized energy storages to participate in energy market and regulation market. [Online]. Available: https://arxiv.org/abs/1901.03941?context=math
[17] G. He, Q. Chen, C. Kang *et al*., "Optimal bidding strategy of battery storage in power markets considering performance-based regulation and battery cycle life," *IEEE Transactions on Smart Grid*, vol. 7, no. 5, pp. 2359-2367, Sept. 2016.
[18] A. Sadeghi and H. Mohsenian-Rad, "Strategic selection of capacity and mileage bids in california ISO performance-based regulation market," in *Proceedings of 2016 IEEE PES General Meeting*, Boston, USA, Jul. 2016, pp. 1-9.
[19] A. Sadeghi and H. M. Rad, "Optimal bidding in performance-based regulation markets: an MPEC analysis with system dynamics," *IEEE Transactions on Power Systems*, vol. 32, no. 2, pp. 1282-1292, Mar. 2016.
[20] P. Kundur, N. J. Balu, and M. G. Lauby, *Power System Stability and Control*. New York: McGraw-hill, 1994.
[21] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge: Cambridge University Press, 2014.
[22] A. Ehrenmann, "Equilibrium problems with equilibrium constraints and their application to electricity markets," Ph.D. dissertation, Fitzwilliam College, Cambridge, 2004.

Mohammad Rayati received the Ph. D. degree in electrical engineering from Sharif University of Technology, Tehran, Iran, in 2019. He is currently with the Center of Excellence in Power System Management and Control, Department of Electrical Engineering, Sharif University of Technology. His current research interests include control and operation of smart grids, power system optimization problems, applications of game theory in electricity markets, and power system dynamics.

Aras Sheikhi received the B.Sc., M.Sc., and Ph.D. degrees in electrical engineering from the Sharif University of Technology, Tehran, Iran, in 2009, 2011, and 2016, respectively. He is currently a Faculty Member of Sharif Energy Research Institute, Tehran, Iran. He is the Director of Smart Sustainable Systems Research Center at Sharif University of Technology. His research interests include sustainable energy systems, industry 4.0 issues in energy systems and energy market.

Ali Mohammad Ranjbar received the M.Sc. degree in electrical engineering from Tehran University, Tehran, Iran, in 1967, and the Ph.D. degree in electrical engineering from Imperial College of London, London, U.K., in 1975. Since then, he has been with the Department of Electrical Engineering, Sharif University of Technology, Tehran, Iran, where he is currently a full Professor. His research interests include power system protection and operation, electrical machines, and smart grid.

Wei Sun received the Ph. D. degree in electrical engineering from Iowa State University, Ames, USA, in 2011. He is currently an Associate Professor in the Department of Electrical and Computer Engineering at the University of Central Florida, Orlando, USA. He is also the Director of Siemens Digital Grid Lab. His research interests include power system restoration and self-healing, and cyber-physical security and resilience.