# Market Power of Coordinated Hydro-wind Joint Bidding: Croatian Power System Case Study

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***Abstract*—The paper analyses the coordinated hydro-wind power generation considering joint bidding in the electricity market. The impact of mutual bidding strategies on market prices, traded volumes, and revenues has been quantified. The coordination assumes that hydro power generation is scheduled mainly to compensate the differences between actual and planned wind power outputs. The potential of this coordination in achieving and utilizing of market power is explored. The market equilibrium of asymmetric generation companies is analyzed using a game theory approach. The assumed market situation is imperfect competition and non-cooperative game. A numerical approximation of the asymmetric supply function equilibrium is used to model this game. An introduced novelty is the application of an asymmetric supply function equilibrium approximation for coordinated hydro-wind power generation. The model is tested using real input data from the Croatian power system.**

***Index Terms*—Asymmetric firm, bidding strategy, coordination, hydro power, wind power, imperfect competition, market power, non-cooperative game, optimal bidding strategy, supply function equilibrium**.

## NOMENCLACURE

| | |
|---|---|
| $\varepsilon$ | Demand shock |
| $\varepsilon_{\max}$, $\varepsilon_{\min}$ | The minimum and maximum possible values of $\varepsilon$ |
| $\varepsilon_k$ | Demand shock, which is the demand value of the $k^{\text{th}}$ outcome of some random variables defined for stochastic demand on probability space $(\Omega_\varepsilon, \mathcal{F}_\varepsilon, P_\varepsilon)$, $\varepsilon_k \in \mathbb{R}_+$ |
| $\beta_{i,k}$ | Slope of linear segment of supply curve at equilibrium price $\pi_k$, $\beta_{i,k} < \infty$ |
| $\xi_{i,k}$ | Ancillary variable used to avoid strict inequalities which enable $\tilde{\pi}_{i,k}$ to fall strictly within $(\pi_k, \pi_{k+1})$, $0.0001 \le \xi_{i,k} \le 0.9999$ |
| $\kappa_{i,k}^{Tu}$ | Shadow price related to the limits of maximum output of company at the $k^{\text{th}}$ residual demand outcome, $\kappa_{i,k}^{Tu} \in \mathbb{R}^+$ |
| $\rho$ | Right-hand side value of inequality constraints used for relaxation of the problem |
| $\pi$ | Market price |
| $[\underline{\pi}, \bar{\pi}]$ | Electricity price range in analyzed market, $\forall \pi \in [\underline{\pi}, \bar{\pi}] \subset \mathbb{R}$ |
| $\pi_k$ | Equilibrium price for the $k^{\text{th}}$ residual demand outcome, $\pi_k \in [\underline{\pi}, \bar{\pi}] \subset \mathbb{R}$ |
| $\tilde{\pi}_{i,k}$ | Supply curve break point for a firm $i$ $\tilde{\pi}_{i,k} \in (\pi_{i,k}, \pi_{i,k+1})$ |
| $\{1, 2, \ldots, I\}$ | Index set of electricity generation firms participating in the electricity market, $i \in \{1, 2, \ldots, I\}$ |
| $\{1, 2, \ldots, K\}$ | Index set of possible demand shocks, i. e., number of possible outcomes, $k \in \{1, 2, \ldots, K\}$ |
| $c_i(y_i)$ | Total short-run cost of electricity generation for a firm $i$ at the output $y_i$, $y_i \in \mathbb{R}_+$ |
| $c_i'(y_i)$ | Short-run marginal cost of the electricity generation for a firm $i$ at the output $y_i$, $y_i \in \mathbb{R}_+$ |
| $D$ | Deterministic and price-insensitive part of the total demand, $D \in \mathbb{R}_+$ |
| $D(\varepsilon_k)$ | Total electricity demand consists of $D$ and $\varepsilon_k$, $D(\varepsilon_k) = D + \varepsilon_k$ |
| $D_r(\pi_k, \varepsilon_k)$ | Residual demand curve for the $k^{\text{th}}$ outcome, which is obtained by subtracting the fringe supply function $s_f(\pi_k)$ from $D(\varepsilon_k)$ |
| $D(\pi)$ | Price-sensitive part |
| $k_{Tu,i}$, $n_{Tu,i}$ | The maximum and minimum power outputs of firm $i$, $k_{Tu,i} \in \mathbb{R}_+$ and $n_{Tu,i} \in \mathbb{R}_+$ |
| $s_i$ | Supply or bidding curve of firm $i$, $s_i$: $[\underline{\pi}, \bar{\pi}] \rightarrow [n_{Tu,i}, k_{Tu,i}]$ |
| $s_f$ | Fringe supply curve |

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$y_{i,k}$ Power output of firm $i$ for the $k^{\text{th}}$ demand shock outcome

$v_{i,k}^{Tu}$ Shadow price related to constraints of company's minimum power output at the $k^{\text{th}}$ residual demand outcome for a firm $i$, $v_{i,k}^{Tu} \in \mathbb{R}^+$

## I. Introduction

WHEN a hydro power plant is scheduled to balance the differences between actual and planned wind power outputs, it is possible to coordinate hydro-wind participation in the electricity market. This coordination may have the opportunity to achieve and utilize market power by choosing optimal bidding strategies. We examine this phenomenon and the impact of these decisions on other electricity market participants. It is shown that some beneficial operation strategies are emerging from short-run coordinated hydro-wind power generation. These advantages are reflected in lower, short-run marginal costs (SRMCs) of hydro power generation [1]. Consequently, this implies that the joint SRMC curves of coordinated hydro-wind power generation are more competitive [2]. We also explore the possibilities for potential synergies that may arise from this joint bidding. Similar operational synergies of different units or in different markets are studied in [3], [4]. These joint bidding strategies mainly arise from low marginal costs of hydro and wind generation on one hand and operational synergies on the other hand. All of this implies that the coordination has the potential to exploit market power to increase market revenue. An example of exerting market power by using capacity restraint is analyzed in [5].

Typically, bidding strategies and curves are based on the skills and expertise of traders and operators of the power units. The research works dealing with the systematic analysis and creation of optimal bidding strategies are extensive. They mainly deal with optimality conditions intended for symmetric solutions (firms are identical competitors and offer the same supply curve) or imply price-taker participants. There are also examples where both price-maker roles (in the day-ahead market) and price-taker roles (balancing market) are studied [6]. A mixed approach assuming both symmetric supply function equilibrium (SFE) and asymmetric equilibria can be found in [7]. Challenges remain for models with asymmetrical firms and different cost functions [8], [9]. An overview of mathematical models of imperfect competition in electricity markets can be found in [10], [11].

In this paper, the focus is on the electricity market state of imperfect competition and non-cooperative game. For this purpose, game theory is utilized and SFE is used because it is considered suitable for modelling competition in electricity markets. Compared with Cournot's models of competition, SFE is more suitable for electricity markets as it enables the competition with supply curves (pairs of volumes and prices) rather than just with volumes. References [12] - [16] present the Cournot models. References [7] and [17]-[19] present the SFE in the electricity market. A comparison of the Cournot and SFE model is given in [20], where a test results indicate the model which best suits the German electricity market of oligopolistic interaction [21]. In [22], SFE is applied to the case of an identical convex cost function and concave demand curves. This is the first comprehensive analysis of SFE, showing the existence of equilibrium in the case of uncertain elastic demand. This enabled the firm to create supply curves with uncertain demand, which is a case in reality. This case is not applicable to electricity markets as they mostly consist of asymmetric firms and short-term inelastic demand.

The first case for solving the issue of short-term inelastic demand in SFE is found in [23]. The same author considers two aspects in [24]. Reference [23] presents the potential impossibility of continuous equilibrium while an infinite set of discontinuous equilibria exists. Reference [24] presents the convergence to a linear equilibrium through learning in a linear supply system. Consequently, in [25], it is shown that there is a unique solution of SFE in the case of inelastic demand and symmetric firms. The first step towards the asymmetric case is made in [26] where the simplest asymmetric firm problem is addressed and the SRMC functions are identical but with different capacity constraints. Another important step towards asymmetric SFE can be found in [27], where the constant SRMC function framework is discussed. Another example of an SFE-based model for utilizing market power by generation withholding can be found in [28]. Although SFE is significantly used in the analysis of the electricity markets [29], the research from [17] makes a real breakthrough in SFE in the analysis of the electricity markets because it solves two main problems. The first one is finding an equilibrium solution in SFE problems, numerically or analytically, even when it is known to exist. The second one is the problem of many possible equilibrium solutions, which results in the successive problem of choosing a certain equilibrium solution. Reference [17] proves that firms with capacity constraints in most cases have a unique equilibrium solution. If there is a family of equilibrium solutions, then they form an ordered family. Due to the elimination of these problems, SFE has become a suitable tool for modelling the competition in the electricity markets. In [30], the objective function regarding constraints on generation capacity and price cap based on a monotonically increasing unique supply curve is studied. The goal is to identify the Nash supply function equilibrium that maximizes the profit of the firms. The competition in the electricity market in England and Wales is analyzed by applying the Nash equilibrium in [31]. The same author in [32] analyzes the issue of increased competition in the electricity market in England and Wales with SFE, i. e., with linear supply functions of asymmetric firms. In [33], he also analyzes another example of SFE with linear supply functions in the contract electricity market in England and Wales. A similar approach of SFE is also applied in [34] and [35] in the case of electricity markets and mixed duopoly. Market power has traditionally been evaluated by Herfindahl-Hirschman index (HHI) and Lerner index (LI) [36]. HHI estimates market power with market concentration measures [37] - [40], while LI measures the deviation of the price from the marginal cost of a firm [41]. In [42], it is argued that the traditional HHI approach correlates poorly with market prices and proposes a new market power index as residual supply index (RSI), which is also used in [43] and [44].

In this paper, the novelty is the application and adaptation of the asymmetric SFE approximation [17] for coordinated hydro-wind electricity market bidding [2]. It is also suitable for coordinated thermo-wind with slight modifications. Based on this, an original model is developed and used for analyzing the effects from the hydro-wind joint bidding strategy and its market power on the electricity market prices, traded volumes, revenues, and the revenues of competing firms. The model is tested on a real case using the input data of the Croatian power system. The developed procedure and findings could be generally useful whenever the impacts of bidding strategies on market prices, volumes, and revenues need to be quantified.

Also, this paper can provide the support to those who need to analyze and quantify the potential of market power utilization for various types of firms with arbitrarily defined SRMC functions and capacity constraints.

The rest of the papers is organized as follows. The method and case study are presented in Sections II and III, respectively. Section IV provides the conclusion of the paper.

## II. Method

The concept assumes that the producers simultaneously submit supply functions to a uniform-price auction in a one-shot game, which is called day-ahead market. With this concept, it is assumed that hydro and wind power plants are owned by the same firm which jointly bids on the market.

In the market state of perfect competition, an optimal supply curve that firm $i$ sends to the short-term electricity market, e.g., day-ahead market, is an SRMC curve of electricity generation which lies above the average short-term variable cost curve. The supply curve obtained in such a way should not be manipulated since it already guarantees maximal revenue. In the state of imperfect competition, there is a possibility for a firm with significant market share to exercise market power which is achieved by shifting the SRMC curve horizontally and/or vertically. This curve shifting results in an optimal supply curve, which means more revenue for a firm. In this paper, the focus is on strongly optimal supply curves, which are obtained via numerical approximation of an SFE defined as a non-linear optimization problem.

We use strongly optimal supply curves derived from an SFE to determine the possibility of joint bidding strategies and their impact on the other electricity market participants. The electricity market state of imperfect competition and the non-cooperative game are assumed, which are modelled using a numerical approximation of the asymmetric SFE [17].

We firstly examine the basic idea behind the SFE form in (1)-(9) before examining its numerical approximation in (10)-(31). This is a short overview of SFE basics and for a full explanation of SFE, and its approximations [17], [18], and [30]. Symbols are defined separately for general form and numerical approximation due to slight differences in indexation in some cases.

The supply function of firm $i$ is strongly optimal in response to the supply curves $s_j$, $j \neq i$ of competing firms. Any realization of demand $D$ provides such output for firm $i$ that ensures the highest possible revenue while considering the generation and price constraints. Equilibrium is called strong equilibrium if all of the supply curves $s_i \, \forall i$ are strongly optimal. It is assumed that firms behave rationally. In other words, they try to guess the expected behavior of competing firms and find the best response. Consequently, they usually end up identifying the strategies of other firms over time, reaching an equilibrium state.

Only strong equilibrium is analyzed here. The supply curves analyzed are only valid for the ranges $[\varepsilon_{\min}, \varepsilon_{\max}]$ and $[\underline{\pi}, \bar{\pi}]$ of demand shock $\varepsilon$ and market prices $\pi$, respectively. In strong equilibrium, the supply curves of a firm $i$ are pointwise and always lie above its SRMC function, which means that no supply is offered at prices below marginal cost. Refer to Lemma 1 and related proof in [17] for more information on this issue.

We assume an oligopolistic electricity market with a maximum of $I$ competing firms where each firm $i$ has the maximum and minimum output capacity $k_{Tu,i} \in \mathbb{R}_+$ and $n_{Tu,i} \in \mathbb{R}_+$ where $i \in \{1, 2, \ldots, I\}$. It is assumed that $c_i$ is a convex and differentiable function that has a bounded first derivative $c_i'(y_i) < \infty$, and is expressed in monetary units (MUs) per MWh. Another assumption is that the SRMC function $c_i'(y_i) \geq 0$.

The electricity demand function is in the form $D(\pi, \varepsilon) = D(\pi) + \varepsilon$. The price-sensitive part $D(\pi)$ is a strictly decreasing, smooth, and concave function. The positive density function $\varepsilon$ is $f(x) > 0$ for $x \in (\varepsilon_{\min}, \varepsilon_{\max})$. The supply curves $s_i$: $[\underline{\pi}, \bar{\pi}] \rightarrow [n_{Tu,i}, k_{Tu,i}]$, as shown in Fig. 1, are defined over the possible market prices $[\underline{\pi}, \bar{\pi}]$, which are mapped to the operation range of firm $i$.

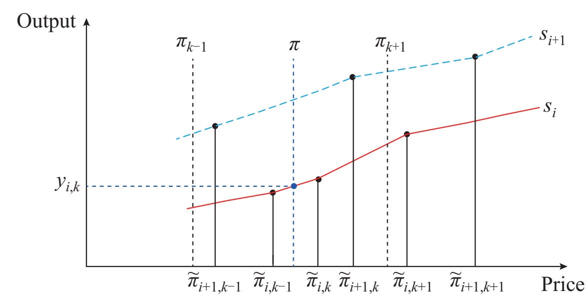


Fig. 1. Hypothetical supply function and its elements.

In SFE, after all firms have chosen their optimal supply curves $s_i$, the market is cleared for demand $D(\pi^*) + \varepsilon^*$, where * denotes the optimal/equilibrium solution. Therefore, the market equilibrium constraint $D(\pi^*, \varepsilon^*) = D(\pi^*) + \varepsilon^* = \sum_{\forall i \in I} s_i(\pi^*)$ clears the market at the price $\pi^*$ and volume $D(\pi^*, \varepsilon^*)$ and gives the unit dispatch by using the right-hand side expression $\sum_{\forall i \in I} s_i(\pi^*)$. Therefore, each firm supplies the amount $s_i(\pi^*)$ and earns a profit of $\pi^* y_i(\pi^*) - c_i(y_i(\pi^*))$. The hardest obstacle in SFE is finding the optimal supply curves $s_i$. The following steps are necessary to obtain $s_i$: ① set up the profit maximization problem of each firm $i$ as shown in (1)-(6). Supply curves obtained in this way are not strongly optimal since they do not account for the optimal behavior of other firms; ② define the problem in (1)-(7) for each firm in order to account for the competing behavior of other firms; ③ derive the profit function (2) of each firm by a price variable as shown in (8) in order to find the strongly optimal supply curves. In this way, the family of optimality conditions is obtained, which consists of $I$ optimality conditions, one for each firm $i$ as shown in (9); ④ solve the family of optimality conditions, i.e., (9) for all firms, simultaneously. In this way, optimality conditions, also called best response functions, are changed to strongly optimal supply functions, which are needed to determine equilibrium prices, volumes, and optimal dispatch. The SFE basics are explained by these four steps. Note that it is suitable to index market prices with index set $i \in \{1, 2, \ldots, I\}$, $\pi_i$. The reason is that all firms are price-makers and can influence equilibrium market prices since they all manipulate the same market price, as (7) is assumed.

$$(k_{Tu,i}, n_{Tu,i}; \bar{\pi}, \underline{\pi}) \in \mathbb{R}_+^2 \times \mathbb{R}^2 \tag{1}$$

$$\max\left(\pi y_i - c_i(y_i)\right) \tag{2}$$

$$\pi \in \mathbb{R} \tag{3}$$

s.t.

$$\bar{\pi} \le \pi \le \underline{\pi} \tag{4}$$

$$n_{Tu,i} \le y_i \le k_{Tu,i} \tag{5}$$

$$y_i := D(\pi) + \varepsilon - \sum_{j \ne i} s_j(\pi) \tag{6}$$

$$\pi_1 = \ldots = \pi_i = \ldots = \pi_I = \pi \tag{7}$$

To put it simply, the family of optimality conditions defined with (9) for $\forall i \in \{1, 2, \ldots, I\}$ is solved simultaneously to obtain equilibrium solution in which these optimality conditions become strongly optimal supply curves.

$$\frac{\partial \Pi_i(\pi, \varepsilon)}{\partial \pi} = 0 \tag{8}$$

$$s_i(\pi) = \left[\pi - c_i'\left(s_i(\pi)\right)\right]\left[\sum_{j \ne i} s_j'(\pi) - D'(\pi)\right] \tag{9}$$

Note that with (1)-(7), the problem is formulated for one demand shock $\varepsilon$. Also, this formulation needs to be approximated in order to be ready for computer implementation [17].

The procedure used to quantify the impact of joint bidding strategies and market power of the coordinated hydro-wind generation on electricity market prices, volumes and competing firms is the optimization problem. Final optimal values of decision variables from the optimization are consequently used to construct supply curves with (10)-(12).

$$\pi_k < \pi_{k+1} \quad \forall k \tag{10}$$

$$y_{i,k} < y_{i,k+1} \quad \forall k \tag{11}$$

$$s_i(\pi) = \begin{cases} y_{i1} + \beta_{i1}\left(\pi - \pi_1\right) & \underline{\pi} \le \pi \le \tilde{\pi}_{i1} \\ y_{i,k} + \beta_{i,k}\left(\pi - \pi_k\right) & \tilde{\pi}_{i,k-1} \le \pi \le \tilde{\pi}_{i,k},\ k = 2, 3, \ldots, K-1 \\ y_{i,K} + \beta_{i,K}\left(\pi - \pi_K\right) & \tilde{\pi}_{i,K-1} \le \pi \le \bar{\pi} \end{cases} \tag{12}$$

The requirements in (10) and (11) ensure that the supply function in (12) is non-decreasing. The supply function in (12) consists of the pair $(y_{i,k}, \pi_k)$ defined for all demand shocks $\forall k \in \{1, 2, \ldots, K\}$.

In the final step, supply curves, revenues, and market prices are plotted to visualize the effect of joint hydro-wind bidding for different installed capacities and short-run marginal costs of competing firms.

In order to explain the whole procedure, the optimization problem set by (13)-(31), which represents a numerical approximation of SFE [17] used for the construction of the curve, is first explained.

We examine firm $i$ and find the one point on its supply curve $s_i$ as shown in Fig. 1. It is necessary to calculate the optimal power output $y_{i,k}$ and the optimal supply price $\pi_{i,k}$ for the $k^{\text{th}}$ scenario of aggregate demand $D_r(\pi_k, \varepsilon_k)$ that can be realized in the electricity market for the identified supply curves of other competitors, $s_j$, $\forall j \ne i$. $y_{i,k}$ is the generation output at the $k^{\text{th}}$ outcome of the residual aggregated demand $D_r(\pi_k, \varepsilon_k)$. In this way, pair $(\pi_{i,k}, y_{i,k})$ is obtained which is one point on the supply curve $s_i$. Since we need more than one point to create a supply curve, we will calculate $K$ pairs $\left\{(\pi_{i,k}, y_{i,k}) \middle| k \in \{1, 2, \ldots, K\}\right\}$ that form $s_i$ of firm $i$. This is done by repeating the calculation for $K$ times for different electricity demand scenarios. We will explain how these electricity demand scenarios are formed. Besides these pairs, $\tilde{\pi}_{ik}$ for scenario $k$ and the slopes of the linear segments connecting the two breaking points, as shown in Fig. 1, are also calculated. In this way, a family of $K$ quadruplets $(\pi_{i,k}, \tilde{\pi}_{i,k}, y_{i,k}, \beta_{i,k})$ is obtained, which is used in (10)-(12) to construct a final supply curve called the strongly optimal supply curve $s_i$ of firm $i$. Since there are $I$ competing firms with different $c_i(y_{i,k})$ and operation ranges, the calculation is simultaneously computed for all of the competing firms. In this way, we crate for each firm one strongly optimal supply curve $s_i$, $\forall i \in \{1, 2, \ldots, I\}$, which forms a strong equilibrium. To increase their expected revenue in this non-cooperative game, firms intend to manipulate market prices and volumes by changing their supply price $\pi_{ik}$, which converges to the single price $\pi_k$ called equilibrium price due to strong equilibrium. Therefore, the indexation $i$ can be omitted from the offer price symbol of each firm, i.e., $\pi_{ik} = \pi_{jk} = \ldots = \pi_{Ik} = \pi_k$. We assume the equilibrium price in this non-cooperative game with imperfect competition equivalent to the market-clearing price in the real electricity market.

The rest of the section explains the procedure in detail. In an optimization problem defined by (13)-(31), let each firm $i$ have a maximum and a minimum outputs $k_{Tu,i} \in \mathbb{R}_+$ and $n_{Tu,i} \in \mathbb{R}_+$ defined by (23), where $i \in \{1, 2, \ldots, I\}$. $\pi_k$ is in the time interval $[0, T]$ in monetary units (MUs) per MWh for $D_r(\pi_k, \varepsilon_k)$ in the electricity market. This residual aggregate demand is obtained by subtracting the elastic fringe supply curve $s_f$ from $D(\varepsilon_k)$ in (31), which is perfectly inelastic. The final demand $D_r(\pi_k, \varepsilon_k)$ in (32) is called residual as it is residual when competitive fringe supply is subtracted from it. This also enables it to become elastic, which is a necessary condition for the strong equilibrium defined in Lemma 1 in [17]. The lemma states that in order to achieve a strong equilibrium, the demand curve must be strictly decreasing, smooth, and concave.

The inelastic part $D(\varepsilon_k)$ in (31) consists of the deterministic part $D$ and the stochastic part $\varepsilon_k$. For the deterministic part $D$, a single forecasted or expected value of the demand for the analyzed period can be used. This part is perfectly inelastic. The second part is inelastic and stochastic. It is called the demand shock and is noted with $\varepsilon_k$ where $k$ indicates the outcome number. With a cumulative distribution function, $F_\varepsilon(\varepsilon)$, on which demand shocks can be sampled using $\varepsilon_k = F_\varepsilon^{-1}(k-1)/(K-1)$ with positive density function $f(x)>0$, $x \in (\varepsilon_{\min}, \varepsilon_{\max})$. With respect to Theorem 2 in [17], since strong equilibrium is required here, the results are independent of the distribution function and the probability of occurrence of each $\varepsilon_k$, which greatly simplifies the procedure, and the probabilities are omitted. The discretization over the demand range $[\varepsilon_{\min}, \varepsilon_{\max}]$ is performed with (28) and is indexed with $\{1, 2, \ldots, K\}$, where the first demand shock is $\varepsilon_1 = \varepsilon_{\min}$ and the last is $\varepsilon_K = \varepsilon_{\max}$. Also, the inequality $\varepsilon_1 \le \varepsilon_2 \le \ldots \le \varepsilon_k \le \ldots \le \varepsilon_K$ should be maintained.

In continuation, the elastic part of the residual aggregated demand in (31) is discussed, which is called the competitive fringe supply function $s_f$. A competitive fringe firm is a large price-taking firm which aggregates all of the small, non-cooperative, price-taking firms competing against each other, which independently have an insignificant market share and must constantly adjust their market position to the position of dominant firms. Although each small price-taking fringe firm has a small market share, all fringe firms together have a significant market share. Therefore, we aggregate them in $s_f$.

It is assumed that the import of electricity to Croatia from Hungary and Slovenia is a competitive fringe with $s_f$. In the short term, electricity demand is almost completely inelastic. Hence, $s_f$ will introduce elasticity into the demand by (31), thus, a necessary condition for strong equilibrium defined in Lemma 1 in [17] will be retained. This residual demand in (31) is then balanced by the dominant firms in the model by (17).

The assumption is that the first derivative of $c_i(y_{i,k})$ defined by (29) is always positive as $c_i'(y_i) \ge 0$, which is ensured by its convexity in the operation range $[n_{Tu,i}, k_{Tu,i}]$. These costs are ordered according to $c_1'(n_{Tu,1}) < c_2'(n_{Tu,2}) < \ldots < c_N'(n_{Tu,I})$ (refer to [17] for more about SFE conditions).

As shown in Fig. 1, $s_i$ needs to be positive, monotonous, and piecewise smooth, which is the case in real markets.

$(\pi_{i,k}, y_{i,k})$ and $\tilde{\pi}_{ik}$ of each linear segment are calculated by (16)-(27) for all outcomes of residual demand, i.e., $D_r(\pi_k, \varepsilon_k)$, $\forall k$. Therefore, the $k^{\text{th}}$ linear segment passes through point $(\pi_{i,k}, y_{i,k})$ with $\beta_{i,k}$. Neighboring linear segments $k$ and $k+1$ will always strictly intersect somewhere between price points $(\pi_{i,k}, \pi_{i,k+1})$, i. e., $\pi_{i,k} < \tilde{\pi}_{i,k} < \pi_{i,k+1}, \forall\, i,k$, which is ensured by (18) and (21). The requirement for strictness assures that the linear segments are smooth functions and the non-differentiable break (intersect) points $\tilde{\pi}_{i,k}$ do not belong to the linear segments.

A brief overview of the SFE approximation [17] used for the coordinated hydro-wind generation is presented in (13)-(31) and modified using $s_f$ and $D_r$.

By minimizing the objective function (14), a quadruple $(\pi_k, \tilde{\pi}_{i,k}, y_{i,k}, \beta_{i,k})$ in (15) is obtained, which characterizes the supply function in (12).

Given $\rho \to 0^+$ and

$$\left(I, K; (c_i)_{i \in \{1,2,\ldots,I\}}; D; \varepsilon_{\max}, \varepsilon_{\min}; k_{Tu,i}, n_{Tu,i}; \bar{\pi}, \underline{\pi}\right) \tag{13}$$

Minimize:

$$\sum_{\forall i}\sum_{\forall k}(\xi_{i,k} - 0.5)^2 + \sum_{\forall i}\sum_{\forall k}\beta_{i,k} \tag{14}$$

over:

$$(\pi_k, \tilde{\pi}_{i,k}, y_{i,k}, \beta_{i,k}; \varepsilon_{i,k}, \kappa_{i,k}^{Tu}, v_{i,k}^{Tu}, \xi_{i,k}) \quad \forall i, \forall k \tag{15}$$

1) The objective function and asymmetric equilibrium. The minimization of the objective function (14), while satisfying the family of equilibrium optimality conditions defined in (16) and other constraints, results in an asymmetric equilibrium. Consequently, the equilibrium solution that holds for all of them is found. Equations (16)-(27) are defined for all $i$ and $k$, i.e., $\forall i \in \{1, 2, \ldots, I\}$ and $\forall k \in \{1, 2, \ldots, K\}$.

$$y_{i,k} - (\pi_k - c_i'(y_{i,k}))\left(\sum_{j \ne i}\beta_{j,k} - D_r'(\pi_k, \varepsilon_k)\right) + \kappa_{i,k}^{Tu} - v_{i,k}^{Tu} = 0 \tag{16}$$

$$\sum_{i=1}^{I} y_{i,k} = D_r(\pi_k, \varepsilon_k) \tag{17}$$

$$y_{i,k+1} - y_{i,k} - \beta_{i,k+1}\pi_{k+1} + \tilde{\pi}_{i,k}(\beta_{i,k+1} - \beta_{i,k}) + \beta_{i,k}\pi_k = 0 \quad k \ne K \tag{18}$$

$$\kappa_{i,k}^{Tu}(k_{Tu,i} - y_{i,k}) \le \rho \tag{19}$$

$$v_{i,k}^{Tu}(y_{i,k} - n_{Tu,i}) \le \rho \tag{20}$$

$$\xi_{i,k}\left(\pi_{k+1} - \pi_k\right) = \pi_{k+1} - \tilde{\pi}_{i,k} + \rho \quad k \ne K \tag{21}$$

$$\underline{\pi} \le \pi_k \le \bar{\pi} \tag{22}$$

$$n_{Tu,i} \le y_{i,k} \le k_{Tu,i} \tag{23}$$

$$\beta_{i,k} \ge 0 \tag{24}$$

$$\kappa_{i,k}^{Tu} \ge 0 \tag{25}$$

$$v_{i,k}^{Tu} \ge 0 \tag{26}$$

$$0.0001 \le \xi_{i,k} \le 0.9999 \tag{27}$$

2) Handling complementarity constraints. Replacing $\kappa_{i,k}^{Tu}(k_{Tu,i} - y_{i,k}) = 0$ and $v_{i,k}^{Tu}(y_{i,k} - n_{Tu,i}) = 0$ with (19) and (20) helps the non-linear programming (NLP) solver address complementarity constraints and reduce computation time. Therefore, the predefined positive initial value of $\rho$ is scaled down at each iteration $\rho \to 0^+$. In addition, at each iteration, (13)-(31) are solved until the problem is unfeasible. $\rho$ in (21) allows the reduction of computing time [17]. Equations (25) and (26) ensure that the dual variables are positive as needed according to the definition of Lagrangian dual of convex

problems.

3) Handling strict inequalities. To make $\tilde{\pi}_{i,k}$ fall strictly within $(\pi_k, \pi_{k+1})$ and to avoid strict inequalities that violate the standard NLP form, the relation $\pi_k < \tilde{\pi}_{i,k} < \pi_{k+1}$ is replaced by (21), which unfortunately needs another strict inequality $0 < \xi_{ik} < 1$ and is again replaced with (27).

$$\varepsilon_k = \varepsilon_{\min} + \frac{(k-1)(\varepsilon_{\max} - \varepsilon_{\min})}{K-1} \tag{28}$$

$$c_i'(y_{i,k}) = \frac{\partial c_i(y_{i,k})}{\partial y_{i,k}} \tag{29}$$

$$D(\varepsilon_k) := D + \varepsilon_k \tag{30}$$

$$D_r(\pi_k, \varepsilon_k) := D(\varepsilon_k) - s_f(\pi_k) \tag{31}$$

## III. Case Study

Equation (31) forms an NLP problem solved using the COIN-OR IPOPT solver with the general algebraic modelling system (GAMS) Rev 239. COIN-OR IPOPT solver for NLP optimization is chosen since it has provided the best overall performance in terms of result quality and computation time regarding other NLP solvers available in GAMS. The case study is conducted for December 5th, 2013. That week of December 2013 is by expert judgement considered the worst-case scenario from the standpoint of wind forecasting error and natural water inflow in hydro power systems of Croatia. The European regional block consisting of Austria, Slovenia, Hungary, and Croatia has been modelled, as shown in Fig. 2.

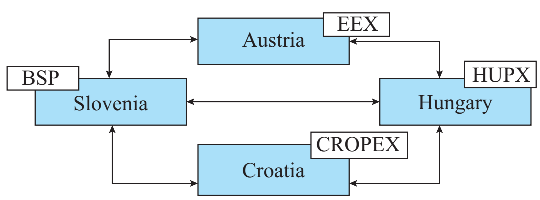


Fig. 2. Depiction of regional block consisting of Austria, Slovenia, Hungary, and Croatia and possible power flows between countries.

The electricity market in Croatia is based on the model of a bilateral market upgraded with the model of balance groups. Further, in 2016, the operation of the Croatian Power Exchange (CROPEX) started. It began with day-ahead trading, and in 2017, intraday trading was introduced, which was an important factor for cost-efficient balancing and the integration of renewable energy sources. In 2013, the total available capacities of all power plants amounted to 4132 MW, of which 1671 MW was for thermal power plants, 2187 MW was for hydro power plants, 254 MW was for wind power plants, and 20 MW was for solar power plants. There was also 348 MW for the nuclear power plant Krško (50% of the total available capacity) in Croatian power system. Typically, the system load ranged from 1100 to 3000 MWh/h [45], and the annual net consumption in 2013 was 16.0 TWh [46]. Also, Croatia had a substantial net cross-border transmission capacity towards Bosnia and Herzegovina, Serbia, Slovenia and Hungary, which in total ranged from about 3000 MW to 4000 MW [47], i.e., usually during peak load.

We assume an oligopoly in the electricity market, where the following firms are analyzed: ① the coordinated hydro-wind generation of the hydro power system Vinodol and wind farm Vrataruša in Croatia; ② the dominant firm representing a former monopoly; ③ competitive fringe which represents the imports from Hungary and Slovenia; and ④ Croatian electricity demand. To simplify the model, the oligopoly is reduced to a duopoly where the coordinated hydro-wind power generation and the dominant firm compete and the bidding strategies are optimized. This is possible since imports are aggregated and presented as a competitive fringe. The installed capacity in the coordinated hydro-wind power generation consists of the Vinodol hydro power plant and the Vrataruša wind power plant. The installed capacity is iteratively scaled up from 132 MW (90 MW of hydro capacity and 42 MW of wind capacity) to 4124 MW. This is done with a constant hydro-wind ratio 68/32 in order to retain initial conditions of the current situation and conduct the analysis of scaling up of coordinated hydro-wind power generation in Croatia, which is also close to the hydro-wind ratio for 2030 defined by the Croatian Energy Strategy for the 2nd scenario of moderate energy transition [48], which is 67/33.

The SRMC function of coordinated hydro-wind and its linear approximation is shown in Fig. 3. For more information on the calculation and construction of SRMCs for the coordinated hydro-wind power generation [1], [2], a method for constructing joint SRMC curves for the coordinated hydro-wind power generation is presented. The stepwise SRMC curve is approximated by a 2nd-degree polynomial approximation that is differentiable and convex to satisfy formal conditions for a strong equilibrium.

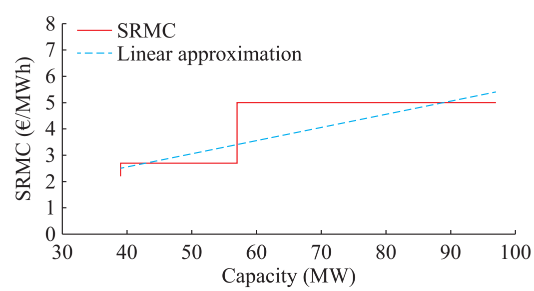


Fig. 3. SRMC function of coordinated hydro-wind and its linear approximation.

Figure 4 shows the dominant firm production portfolio shown in a merit order way and its 2nd-degree polynomial approximation. It is also a differentiable and convex function that satisfies the formal conditions for strong equilibrium. Although these approximations involve the errors in the solution, the errors are tractable and enable convexity, meaning that the global optimum can be easily achieved. It is preferable that the 2nd-degree polynomial approximation has no negative value in the range of 250 MW to 1250 MW, but it does not affect the final solution to a great extent. It reduces the computation time and ensures the convexity of the problem (due to the 2nd polynomial) compared with the exponential function and higher-order polynomials.

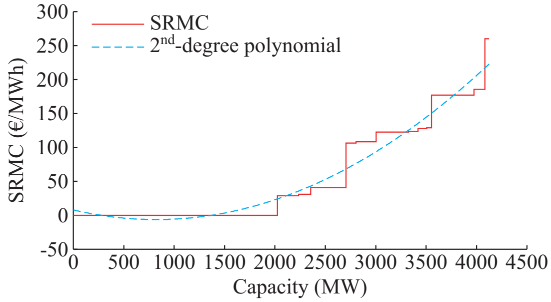


Fig. 4. Dominant firm production portfolio shown in a merit order way and its 2nd-degree polynomial approximation.

Figure 5 shows the function of electricity imports to Croatia from Hungary and from Slovenia as a function of nodal prices in Croatia. These functions represent the competitive $s_f$ of the imports in the model. The aggregate demand function is perfectly inelastic and consists of a deterministic and a stochastic part. The deterministic part $D$ is equal to 2307 MW for all demand shocks, which is an average electricity demand on December 5, 2013. The stochastic part $\varepsilon_k$ is bounded by $\varepsilon_{\min}=-2307$ MW and $\varepsilon_{\max}=300$ MW. $\underline{\pi}$ and $\overline{\pi}$ are set to be −500 €/MWh and 3000 €/MWh according to the CROPEX.

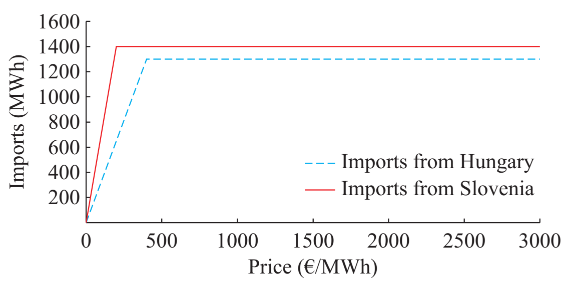


Fig. 5. Function of electricity imports to Croatia from Hungary and from Slovenia as a function of nodal prices in Croatia, obtained by two-stage least squares (2SLS) regression.

## A. Results

The results of the case study are shown in Figs. 6-9. Figure 6(a) shows the coordinated hydro-wind for different hydro-wind capacities. Figure 6(b) shows the dominant firm for different hydro-wind capacities. Each supply curve corresponds to one different level of installed capacity in the coordinated hydro-wind power generation from 132 MW to 4124 MW. In this way, it is possible to assess how a different size of the coordinated hydro-wind affects market prices and volumes.

Figure 7(a) shows strongly optimal supply curves for the coordinated hydro-wind when the SRMC of the coordinated hydro-wind varies from 0 €/MWh to 60 €/MWh, while the hydro-wind capacity is fixed at 132 MW. In these situations, the supply curves of the dominant firm are also shown in Fig. 7(b), which evaluates how different production costs of the coordinated hydro-wind affect market prices and volumes.

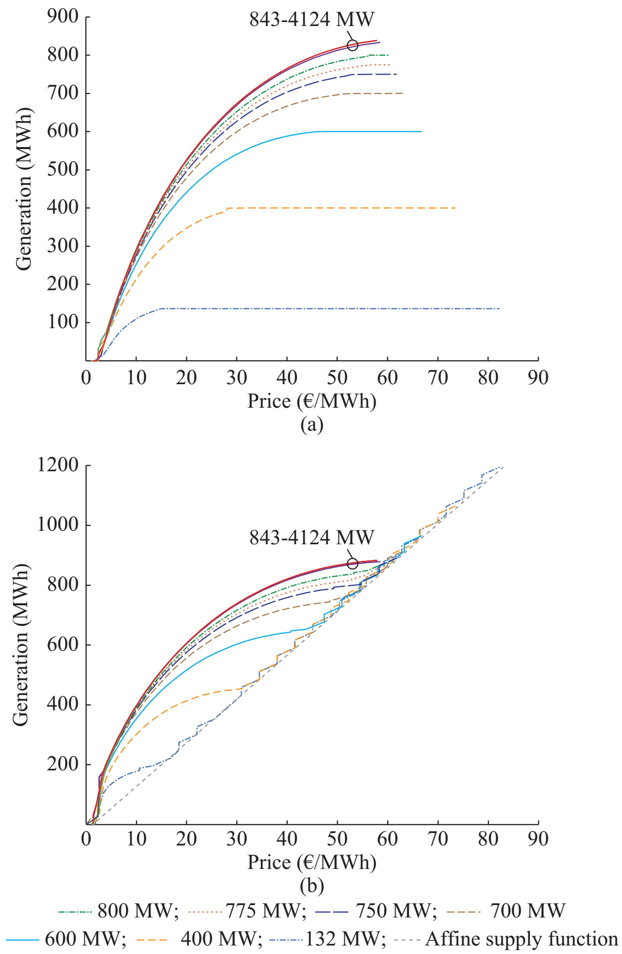


Fig. 6. Strongly optimal supply curves. (a) Coordinated hydro-wind for different hydro-wind capacities. (b) Dominant firm for different hydro-wind capacities.

In Fig. 8, a comparison of market prices is shown for the cases when hydro-wind jointly bids 843 MW of capacity in addition to the case of separate market bidding (non-coordinated generation) with 268 MW of wind and 575 MW of hydro. A comparison is made for different realizations of demand, which quantifies the effects of joint bidding on market prices. It can be observed that the optimal bidding of coordination tends to smooth the price curve for different demands. Figure 9 shows the revenue curves as a function of electricity demand for joint bidding of coordinated and non-coordinated hydro-wind power generations. The positive aspects of joint bidding of coordinated hydro-wind power generation have been quantified, i.e., the synergy in the form of increased revenues.

## B. Discussion

In Figs. 6 and 7, the supply functions of the coordinated hydro-wind and the dominant firm show some distinctive features as follows.

1) Increasing hydro-wind installed capacity: the convergence of equilibrium solutions. The supply functions of the coordinated hydro-wind strongly converge during the increase of the installed power in the coordinated hydro-wind from 132 MW to 843 MW.

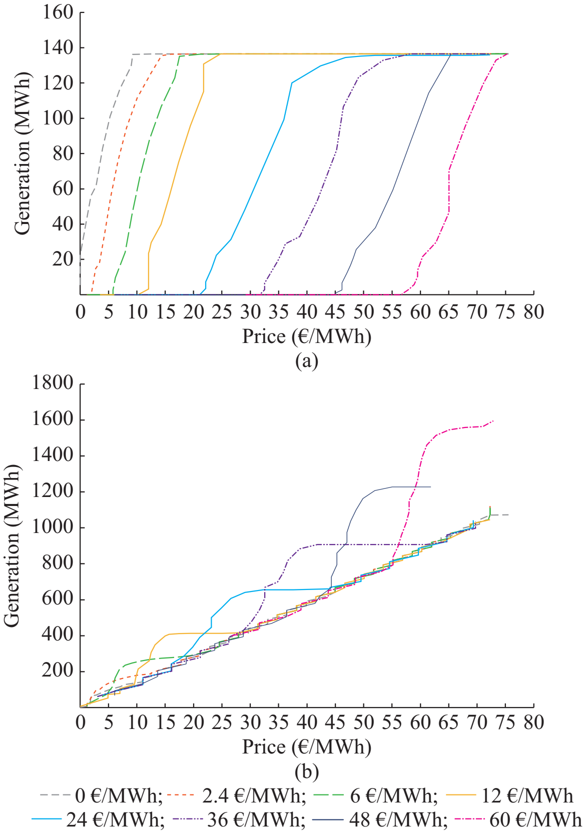


Fig. 7. Strongly optimal supply curves. (a) Coordinated hydro-wind for different SRMC. (b) Dominant firm for different values of coordinated hydro-wind SRMC.

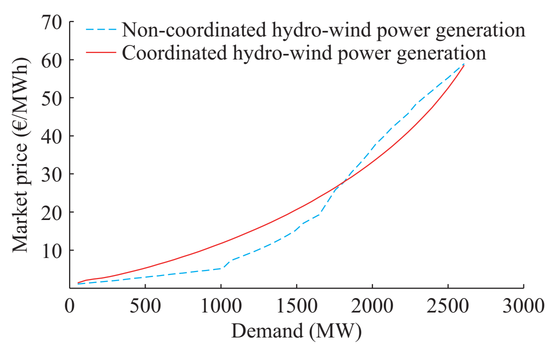


Fig. 8. Market price as a function of electricity demand for joint bidding of coordinated and non-coordinated hydro-wind power generations.

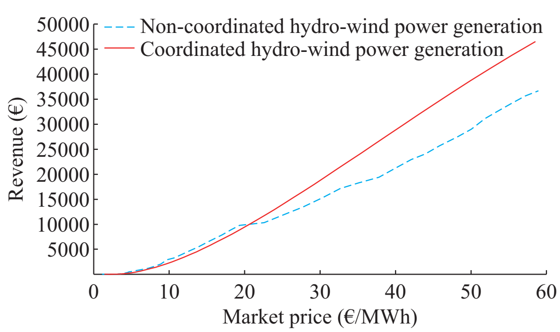


Fig. 9. Revenue curves as a function of electricity demand for joint bidding of coordinated and non-coordinated hydro-wind power generations.

After the 843 MW of capacity in the coordination, the supply curves are practically the same up to 4124 MW, which means that the optimal market strategy for the capacity ranging from 843 MW to 4124 MW is the same. Therefore, there is no need for installing hydro-wind capacity higher than 843 MW, as it will not achieve any growth of market power as shown in Fig. 6(a), where the supply curves from 843 MW to 4124 MW are equal.

2) Increasing hydro-wind installed capacity: the market power of the coordinated hydro-wind. In Fig. 6(b), the supply functions of the dominant firm are shown for the cases where the capacity in coordinated hydro-wind increases from 132 MW to 4124 MW. The figure illustrates that the supply functions of the dominant firm deviate from affine supply function, which represents the supply function of the dominant firm when it has a monopoly as shown in Fig. 6(b), whenever the coordinated hydro-wind is introduced into the electricity market. This deviation also increases in magnitude as the capacity in the coordinated hydro-wind increases.

In the range of 38 MW to 60 MW, the market power of the coordination is negligible and the coordination acts as a competitive fringe. The market power of coordinated hydro-wind becomes visible for capacities ranging from 66 MW to 132 MW. If the installed power in coordination increases more (from 132 MW onwards), then the dominant firm deviates extensively from its monopolistic values. Therefore, at 132 MW, the coordination has limited market power since it is at low market prices (up to 15 €/MWh), which is a rare situation in the analyzed electricity market, as shown in Fig. 6(b). As its installed power increases, it becomes clear that market power of coordinated hydro-wind becomes more significant and can be utilized at a higher price (up to 60 €/MWh) and a higher demand. After achieving a value of 843 MW in coordination, the supply functions of the dominant firm are the same. This means that there is no need for more than 843 MW in the capacity of coordinated hydro-wind since the revenues of coordination would not increase, i.e., the coordination cannot influence the prices above 60 €/MWh and they are the same as in affine supply function. When the installed power is 843 MW, the coordination achieves a market share of 49% at residual demand. The generally accepted empirical values of market power are 50% or more [49], which means that the coordination of 843 MW achieves market power and is the dominant market player.

3) Increasing hydro-wind marginal costs: a horizontal shift in hydro-wind supply functions. Figure 7(a) shows the supply curves of the coordinated hydro-wind for different SRMC (from 0 €/MWh to 60 €/MWh) of the coordinated hydro-wind. It is clear from Fig. 7(a) that the supply curves are above the SRMC. Consequently, the supply curves of the coordinated hydro-wind shift to the right as its SRMC rises. The shifting phenomenon is the result of Lemma 1 from [17], i.e., no supply is offered at the prices lower than the marginal cost. The result of this phenomenon is a decrease in hydro-wind power generation as its SRMC increases.

4) Increasing hydro-wind SRMC: market power of the co-

ordinated hydro-wind. Figure 7(b) shows the dominant firm for different values of coordinated hydro-wind SRMC. It can be observed that the coordinated hydro-wind reduces its generation to increase $\pi_k$, as its SRMC increases, as shown in Fig. 7(a). This is clearly visible in Fig. 7(a) and (b) for one situation of $\pi_k = 40$ €/MWh. At the same time, the dominant firm annuls that behavior by increasing its generation (dominant firm increases the generation from 660 MWh to 906 MWh and prevents a rise of market prices which stay constant at $\pi_k = 40$ €/MWh) as the hydro-wind reduced generation due to SRMC rises from 24 €/MWh to 36 €/MWh. This behavior of the dominant firm is attributed to the fact that hydro-wind withholding generation enables the dominant firm to produce more at the same equilibrium price.

5) Increasing revenues and smoothing market prices. The impact of joint bidding on market prices and revenues for the case of coordinated and non-coordinated hydro-wind power generations is quantified and shown in Figs. 8 and 9, respectively. As expected, Fig. 8 shows that market prices rise as the demand grows in both cases with different magnitudes. It can be observed that the optimal bidding of coordination tends to smooth the price curve for different demands, as shown in Fig. 8, lowering prices at higher demands, increasing prices at lower demands, and annulling the negative effects of low off-peak prices due to wind overproduction when negative pricing is possible. This is the result of joint bidding strategies and a positive effect of joint bidding of coordinated hydro-wind power generation as it annuls the high wind penetration. Since in a coordinated case, excess production is stored in a hydro reservoir. Therefore, wind curtailment is avoided.

Joint bidding of coordinated hydro-wind power generation achieves market power at lower demand levels and increases market prices at higher rates. Due to joint bidding of coordinated hydro-wind generation, it can achieve higher production levels and higher revenues compared with the non-coordinated case, as shown in Fig. 9. In other words, the coordinated hydro-wind power generation is more effective in terms of revenue. At a market price of around 58.9 €/MWh, the joint hydro-wind bidding shows a profit higher by 26.8% compared with non-coordinated bidding. The significant increase in profits is possible in a wide range of market prices. A comparison of the impact of coordinated and non-coordinated hydro-wind power generations on market prices and overall profit provides the justification for the consideration of the coordinated hydro-wind concept in the Croatian case.

## IV. Conclusion

The paper analyses the market behavior of joint bidding of coordinated hydro-wind generation on the electricity market. The coordination assumes that the hydro power generation balances wind forecasting errors. The paper contributes to the analysis of market power and the optimal bidding of coordinated hydro-wind using a numerical approximation of the supply function equilibrium that models the oligopoly in Croatia and imports from Hungary and Slovenia.

This paper can be useful in the cases where the impact of bidding strategies on market prices, volumes, and revenues needs to be quantified. This paper can also help those who need to analyze the possibility of joint bidding of coordinated hydro-wind to achieve market power and the impact of the joint bidding on market prices and volumes.

A comparison of the impact of coordinated and non-coordinated hydro and wind generation on market prices and overall revenue justifies the consideration of the hydro-wind concept in the Croatian case. Specifically, the coordinated hydro-wind generation achieves higher profits and annuls the effects of low off-peak prices caused by wind overproduction in a non-coordinated case. Therefore, "missing money" problems due to large wind penetration are avoided.

Future research works include the application of SFE and game theory for the coordination of the solar photovoltaic systems and batteries [50] in local electricity markets [51]. Furthermore, the potential of wind turbines and coordinated operation with other sources will be studied in order to balance the power system and mitigate active power disturbances [52], [53]. Finally, it must be noted that market coordination between two separately owned power plants is prohibited. Therefore, this paper assumes that hydro and wind power plants are owned by the same entity.

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