

A Multi-parametric Programming Based Analytic Method to Compute Consumer Offer Curve for Reserves

Weiye Zheng, David J. Hill, and Wenchuan Wu

Abstract—An analytic method is proposed to compute the price-reserve offer curve at the consumer level in hierarchical direct load control. The convexification of the consumer reserve provision is examined, and the analytic expression of the optimal solution within each critical region is derived. Then, based on multi-parametric programming, a combinatorial enumeration method in conjunction with efficient reduction and pruning strategy is proposed to compute the optimal response of consumers in the whole price space. Numerical tests along with an application example in the bi-level aggregator pricing problem demonstrate the merit of this method.

Index Terms—Direct load control, multi-parametric programming, price-reserve offer curve, analytic solution.

I. INTRODUCTION

WITH the integration of an increasing amount of renewable sources, power systems need more reserve capacity to keep power balance. As demand response has been recognized as an effective and convenient way for energy management [1], [2], responsive loads are utilized by instituted programs in several electricity markets for reserve provision. But the control strategy is relatively crude in most cases. To achieve fine control, a mathematical method is necessary. Since the third party aggregators are entering the reserve market, direct load control (DLC) involves the interaction between aggregators and consumers, which is a bi-level optimization problem. Then, one of the most fundamental issues is how to characterize the optimal response of the lower level, i.e., the reserve provided by consumers with a certain price signal. Substituting the Karush-Kuhn-Tucker

(KKT) conditions of the lower-level problem into the upper-level problem, which is also known as mathematical programming with equilibrium constraints (mpECs), is a prevailing method [3]. However, serious privacy concerns are raised since this practice requires the aggregator to collect the full model from all consumers [4]. This issue could be alleviated by a probing method (PM) [5], where the aggregator sends a certain price to the consumers, observes their offered reserve capacity, and iterates until a stepwise constant price-reserve estimate for each consumer is developed. However, an accurate estimate requires a large number of probing events and communications, which is impractical for real-time DLC that requires both the accuracy and efficiency.

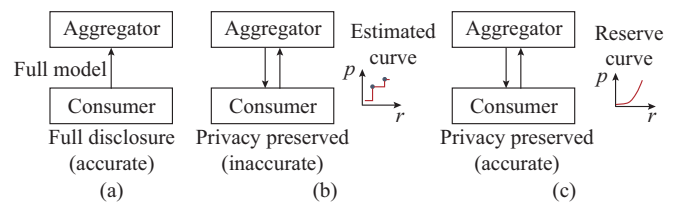


Fig. 1. Comparison of three methods to characterize consumers' response in DLC. (a) mpEC. (b) PM. (c) Proposed analytic method.

To tackle these issues, an efficient method is proposed to obtain the optimal price-reserve offer curve [6] in an analytic form. The idea is inspired by network equivalent representation in integrated systems [7], [8]. In the proposed scheme, the aggregator just specifies the price bounds, and then consumers calculate and upload their offer curve to the aggregator for further dispatch. Figure 1 presents the comparison of three methods to characterize consumers' response in DLC.

To achieve this, multi-parametric programming is employed [9], which has also been applied in power system operation such as economic dispatch (ED). However, this letter differs from relevant multi-parametric programming research in both the aim and methodology. Multi-parametric is employed in [10] for decomposition, where only a few critical regions (CRs) are explored to achieve the decomposition in multi-area ED. While the multi-parametric programming based enumeration procedure (MPEP) is investigated along with the convexification technique to compute the optimal reserve provision curve in the whole price space for DLC.

The merits of this short letter are summarized as follows.

1) Most existing literature adopts stepwise constant price-

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reserve offer curves, which may be convenient but not optimal. An MPEP is proposed, for the first time, to accurately describe the optimal reserve provision.

2) Calculating the optimal reserve with a specific price signal has been well studied by the existing works. However, the difficulty is how to obtain the optimal reserve in the whole price space where numerous cases of price signals should be considered. To this end, a reduction and pruning method is proposed to explore all CRs efficiently.

3) A bi-level aggregator pricing (AP) problem is listed as an example to demonstrate the application of the proposed method. Numerical results show that even in this simple case, the economic improvement due to more accurate price-reserve offer curves is still substantial.

II. PROBLEM OF CONSUMER RESERVE PROVISION (CRP)

With a certain $\mathbf{p}_t = (p_{t,up}, p_{t,down})^T$, i.e., the price signal sent from the aggregator to the consumer for up- and down-reserve capacity at time t , the problem of CRP where each consumer determines its optimal up- and down-reserve capacity $r_{t,up}, r_{t,down}$ is formulated as follows for any time t [5]:

$$\max_{r_{t,up}, r_{t,down}} p_{t,up} r_{t,up} + p_{t,down} r_{t,down} - C_{t,shift}(r_{t,shift}) - C_{t,shed}(r_{t,shed}) - C_{t,inc}(r_{t,inc}) \quad (1)$$

s.t.

$$d_t + r_{t,down} \leq \bar{d}_t \quad (2)$$

$$\underline{d}_t + r_{t,up} \leq d_t \quad (3)$$

$$r_{t,shift} = \min(r_{t,up}, r_{t,down}) \quad (4)$$

$$r_{t,shed} = \max(r_{t,up} - r_{t,down}, 0) \quad (5)$$

$$r_{t,inc} = \max(r_{t,down} - r_{t,up}, 0) \quad (6)$$

$$C_{t,shift}(r_{t,shift}) = a_{t,shift} + b_{t,shift} r_{t,shift} + c_{t,shift} r_{t,shift}^2 \quad (7)$$

$$C_{t,shed}(r_{t,shed}) = a_{t,shed} + b_{t,shed} r_{t,shed} + c_{t,shed} r_{t,shed}^2 \quad (8)$$

$$C_{t,inc}(r_{t,inc}) = a_{t,inc} + b_{t,inc} r_{t,inc} + c_{t,inc} r_{t,inc}^2 \quad (9)$$

where d_t , \bar{d}_t , \underline{d}_t are the current, maximum, and minimum power consumptions of the consumer at time t , respectively; $\mathbf{r}_t = (r_{t,shift}, r_{t,shed}, r_{t,inc})^T$ denotes the approximated load shifting, load shedding, and load increasing components of the reserve capacity at time t , respectively; $C_{t,shift}(\cdot)$, $C_{t,shed}(\cdot)$, $C_{t,inc}(\cdot)$ are the corresponding costs, respectively; and $a_{t(\cdot)}$, $b_{t(\cdot)}$, $c_{t(\cdot)}$ are the cost coefficients.

The model can be described as follows. The objective in (1) is the total profit maximization of CRP. Constraints (2) and (3) bound up- and down-reserve by load consumption, respectively. Constraints (4) - (6) factorize the reserve into shifting, shedding, and increasing components, while constraints (7)-(9) represent the corresponding costs.

Note that $r_{t,up} = r_{t,shift} + r_{t,shed}$ and $r_{t,down} = r_{t,shift} + r_{t,inc}$. To avoid non-differential representation of (4)-(6), CRP can be rewritten equivalently as CRP' in the following form [5], which includes (10)-(16) and (7)-(9):

$$\max_{r_{t,shift}, r_{t,shed}, r_{t,inc}} p_{t,up}(r_{t,shift} + r_{t,shed}) + p_{t,down}(r_{t,shift} + r_{t,inc}) - C_{t,shift}(r_{t,shift}) - C_{t,shed}(r_{t,shed}) - C_{t,inc}(r_{t,inc}) \quad (10)$$

s.t.

$$r_{t,shift} + r_{t,inc} \leq \bar{d}_t - d_t \quad \lambda_{t,down} \leq 0 \quad (11)$$

$$r_{t,shift} + r_{t,shed} \leq d_t - \underline{d}_t \quad \lambda_{t,up} \leq 0 \quad (12)$$

$$r_{t,shift} \geq 0 \quad \lambda_{t,1} \geq 0 \quad (13)$$

$$r_{t,shed} \geq 0 \quad \lambda_{t,2} \geq 0 \quad (14)$$

$$r_{t,inc} \geq 0 \quad \lambda_{t,3} \geq 0 \quad (15)$$

$$r_{t,shed} r_{t,inc} = 0 \quad (16)$$

where $\lambda_t = (\lambda_{t,down}, \lambda_{t,up}, \lambda_{t,1}, \lambda_{t,2}, \lambda_{t,3})^T$ is the multipliers of the corresponding constraints under the KKT conditions. Bilinear constraint (16) makes this model strongly nonconvex and hard to solve.

III. MULTI-PARAMETRIC PROGRAMMING BASED ENUMERATION PROCEDURE

CRP' can be written as two convex problems (CRP1 and CRP2) by nulling $r_{t,shed}$ and $r_{t,inc}$, respectively, which provides two solutions of reserve provision. The final decision made by the consumer will be the solution that brings more profits. However, this procedure will involve solving two problems. To further speed up the solution, the condition where constraint (16) can be relaxed is discussed, and the analytic price-reserve offer curve among the whole investigated price space $\mathbf{P}_t = \{\mathbf{p}_t\}$ can be obtained via solving one convex problem instead of two. rCRP is denoted as the convex relaxed version of CRP by removing (16).

Proposition 1: the optimal solution of rCRP is denoted as $\mathbf{r}_t^* = [r_{t,shift}^*, r_{t,shed}^*, r_{t,inc}^*]^T$. Then relaxing (16) is exact under the following sufficient conditions with regard to the marginal costs, where $C'_{(\cdot)}$ is the first-order derivative of the cost function $C_{(\cdot)}$.

- 1) Condition 1: $C'_{t,shift}(0) < C'_{t,shed}(r_{t,shed}^*) + C'_{t,inc}(r_{t,inc}^*)$.
- 2) Condition 2: $C'_{t,shift}(r_{t,shift}^*) \neq C'_{t,shed}(r_{t,shed}^*) + C'_{t,inc}(r_{t,inc}^*)$.

Before proving Proposition 1, a lemma is stated as follows.

Lemma 1: if there exists $r_{t,shed}^* > 0$ and $r_{t,inc}^* > 0$ in the optimal solution of rCRP, then $r_{t,shift}^* > 0$ under condition 1.

Proof of Lemma 1: assume $r_{t,shift}^* = 0$. With a small enough positive ε , $\delta = \min(\varepsilon, r_{t,shed}^*, r_{t,inc}^*)$ is set, and it follows that $\mathbf{r}'_t = (\delta, r_{t,shed}^* - \delta, r_{t,inc}^* - \delta)^T$ is also a feasible solution of rCRP. The difference of corresponding objectives between solutions \mathbf{r}'_t and \mathbf{r}_t^* is $[C'_{t,shed}(r_{t,shed}^*) + C'_{t,inc}(r_{t,inc}^*) - C'_{t,shift}(0)]\delta + o(\delta)$, which is positive under condition 1, and $o(\delta)$ is an infinitesimal of higher order than δ . This contradicts with the optimality of solution \mathbf{r}_t^* .

Proof of Proposition 1: assume there exists $r_{t,shed}^* > 0$ and $r_{t,inc}^* > 0$ in the optimal solution of rCRP, then we have $r_{t,shift}^* > 0$ under condition 1 according to Lemma 1. Then $\lambda_{t,1} = 0$, $\lambda_{t,2} = 0$, $\lambda_{t,3} = 0$. Let L denote the Lagrangian of rCRP. Using KKT conditions, we can obtain $\partial L / \partial r_{t,shed} = C'_{t,shed}(r_{t,shed}^*) - p_{t,up} - \lambda_{t,up} - \lambda_{t,2} = 0$ and $\partial L / \partial r_{t,inc} = C'_{t,inc}(r_{t,inc}^*) - p_{t,down} - \lambda_{t,down} - \lambda_{t,3} = 0$.

Combining these with the fact that $\lambda_{t,1}, \lambda_{t,2}, \lambda_{t,3} = 0$ yields $\partial L / \partial r_{t,shift} = C'_{t,shift}(r_{t,shift}^*) - (p_{t,up} + p_{t,down}) - (\lambda_{t,up} + \lambda_{t,down}) - \lambda_{t,1} = C'_{t,shift}(r_{t,shift}^*) - C'_{t,shed}(r_{t,shed}^*) - C'_{t,inc}(r_{t,inc}^*) \neq 0$ under condition 2,

which violates KKT conditions. Hence, $r_{i,shed}^* > 0$ and $r_{i,inc}^* > 0$ cannot appear simultaneously in the optimal solution of rCRP. Therefore, the relaxation is exact under sufficient conditions 1 and 2.

Remark: the aim of condition 1 is to avoid zero load shifting at the optimal point. In fact, compared with the load shedding, the cost of load shifting is relatively lower, and is usually prioritized. Conditions 1 and 2 can also be replaced by a stricter condition, which can be checked beforehand.

$$\sup \{C'_{i,shift}\} < \inf \{C'_{i,shed}\} + \inf \{C'_{i,inc}\} \quad (17)$$

Although condition (17) may not hold all the time, CRP1, CRP2, and rCRP are all convex quadratic programming problems, where the following MPEP is applicable to obtain the price-reserve offer curve from these problems. The set of binding constraints is denoted as the active set \mathcal{A} . Without the loss of generality, CRP1, CRP2, and rCRP can be compacted as CPR- \mathcal{A} in the following form:

$$\max_{r_i} \left(p_i^T E^T r_i - \frac{1}{2} r_i^T C_i r_i - b_i^T r_i \right) \quad (18)$$

s.t.

$$B_{\mathcal{A}} r_i = d_{i,\mathcal{A}} \quad \lambda_{i,\mathcal{A}} \leq 0 \quad (19)$$

$$B_{\mathcal{I}} r_i \leq d_{i,\mathcal{I}} \quad \lambda_{i,\mathcal{I}} = 0 \quad (20)$$

where B is a constant coefficient matrix determined by constraints; and d is a corresponding right-hand side vector; a matrix or vector with a subscript \mathcal{I} denotes the sub-matrix or vector associated with active or inactive constraints; $C_i = \text{diag}(2c_{i,shift}, 2c_{i,shed}, 2c_{i,inc})$; $b_i = (b_{i,shift}, b_{i,shed}, b_{i,inc})^T$; and E is a 3×2 constant matrix $[1, 1; 1, 0; 0, 1]$. With a price signal p_i , the optimal response, i.e., reserve provision, from a consumer can be derived in an analytic form:

$$r_{i,\mathcal{A}}^* = (Q_{i,\mathcal{A}} E) p_i + (R_{i,\mathcal{A}}^T d_{i,\mathcal{A}} - Q_{i,\mathcal{A}} b_i) \quad (21)$$

which holds for any p_i in the following CR:

$$\left\{ \begin{array}{l} \lambda_{i,\mathcal{A}} \leq 0 \Rightarrow R_{i,\mathcal{A}}^T E p_i \leq R_{i,\mathcal{A}}^T b_i - S_{i,\mathcal{A}} d_{i,\mathcal{A}} \\ p_i, B_{\mathcal{I}} r_{i,\mathcal{A}}^* \leq d_{i,\mathcal{I}} \Rightarrow A_{\mathcal{I}} Q_{i,\mathcal{A}} E p_i \leq d_{i,\mathcal{I}} - A_{\mathcal{I}} R_{i,\mathcal{A}}^T d_{i,\mathcal{A}} + Q_{i,\mathcal{A}} b_i \\ p_i \in P_i \end{array} \right\} \quad (22)$$

where $S_{i,\mathcal{A}} = -(B_{\mathcal{A}} C_i^{-1} B_{\mathcal{A}}^T)^{-1}$, $R_{i,\mathcal{A}} = -S_{i,\mathcal{A}} B_{\mathcal{A}} C_i^{-1}$, and $Q_{i,\mathcal{A}} = C_i^{-1} - C_i^{-1} B_{\mathcal{A}}^T R_{i,\mathcal{A}}$, which are all constant matrices.

To enumerate all active sets \mathcal{A} , a combinatorial tree listing all possibilities for \mathcal{A} is established as shown in Fig. 2. The reduction and pruning are proposed to further improve the efficiency when exploring different CRs that cover the whole price space P_i . If condition (17) is satisfied, then at least one of (14) and (15) should be active. Active sets denoted by the dashed blue box in Fig. 2 can be removed *a priori*, and the scale of the tree has reduced from 26 sets to 12 sets. The reduced tree is enumerated from level 1. For each candidate \mathcal{A} , the feasibility of (19) and (20) is checked. For instance, if $\{14\}$ is infeasible, all active sets containing $\{14\}$ will be pruned [11], as shown by the red solid box in Fig. 2.

To sum up, Fig. 3 presents the whole flowchart to obtain the price-reserve offer curve using the proposed MPEP method, which shows that the proposed method does not rely on condition (17). If condition (17) holds, the price-reserve of-

fer curve can be readily calculated. Otherwise, the offer curve can be still obtained by solving CRP1 and CRP2, respectively, and comparing their profits.

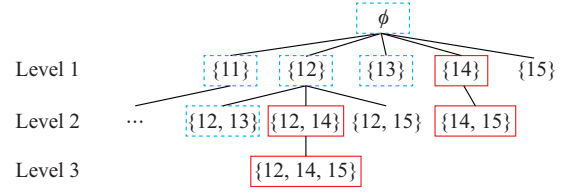


Fig. 2. Combinatorial tree listing all possibilities for MPEP.

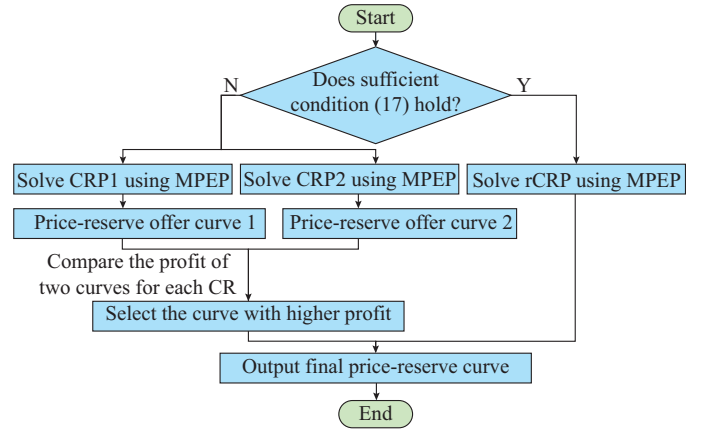


Fig. 3. Whole flowchart to obtain price-reserve offer curve using proposed MPEP method.

IV. SIMULATION RESULT

The parameter setting remains the same as type III consumer in [9], except that $[d_t, \bar{d}_t]$ is set to be $[7.5, 9]$ for $9 \leq t \leq 19$ and $[1.5, 4.0]$ for other time slots, and $b_{i,shift}$ is set to be 50 for a clearer presentation of the price-reserve offer curve; $p_{i,up} \geq 0$ and $p_{i,down} \leq 100$. A time horizon of 24 hours is investigated. For each time t , 10201 benchmark price-reserve samples $(r_{i,up}^{Bench}, r_{i,down}^{Bench}, p_{i,up}^{Bench}, p_{i,down}^{Bench})^T$ uniformly distributed in P_i are obtained by solving CRP repeatedly. PM(n) represents PM with a step of $\Delta p = n$ solved by IPOPT. The error compared with each sample of the benchmark is defined as $\max(|r_{i,up} - r_{i,up}^{Bench}|, |r_{i,down} - r_{i,down}^{Bench}|)$.

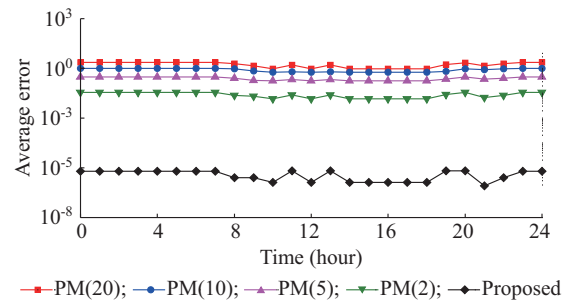


Fig. 4. Comparison of average errors over 10201 samples in evolution of t .

In Fig. 4, the proposed method achieves the most accurate price-reserve offer curve within the 24-hour horizon, which also validates the exactness of the relaxation in Proposition 1. As shown by the detailed comparison at $t = 17$ hour in Ta-

ble I, there must to be a trade-off between the accuracy depending on the resolution and efficiency in PM, while the proposed method is significantly superior in accuracy and efficiency. As shown in Fig. 5, the piece-wise affine price-reserve offer curve is in accordance with (21) and (22). In Fig. 5, up-reserve is not presented due to limited space. When the prices are too low, the consumer will find that it is not profitable to provide reserves, as shown in the CR1. As the prices increases, the optimal reserve is influenced by both $p_{t,sup}$ and $p_{t,down}$. It is intuitive that a higher $p_{t,down}$ encourages more down-reserves, while a higher $p_{t,sup}$ may induce more down-reserves indirectly via the load shifting component. When the prices increase to a certain level, e.g., the CR5, the consumer has reached the physical limit, and the down-reserve will remain at the maximal available value. The curve may help upper-level aggregators with further analysis and operation.

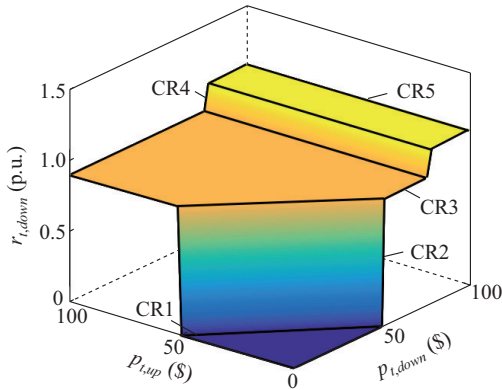


Fig. 5. Price-reserve offer curve ($t = 17$ hour) divided by 5 CRs using proposed MPEP method.

TABLE I
DETAILED COMPARISON OF PRICE-RESERVE CURVE GENERATION USING DIFFERENT METHODS

Method	Resolution	Average error	Time (s)
PM(20)	36	9.85×10^{-2}	0.308
PM(10)	121	7.20×10^{-2}	1.058
PM(5)	441	3.23×10^{-2}	5.335
PM(2)	2601	5.95×10^{-2}	22.169
Proposed		2.54×10^{-2}	0.048

V. APPLICATION IN BI-LEVEL AP PROBLEM

To further demonstrate how the proposed method can be applied to help the upper-level aggregator for decision making, the following problem of bi-level AP [12], including (23)-(26) and (11)-(15), is considered as an illustrating example as:

$$\max_{\{p_{i,t}\}} f(\{p_{i,t}\}, \{r_{i,t}^*(p_{i,t})\}) = \zeta_t^T \sum_{i=1}^N \mathbf{E}^T r_{i,t}^* - p_{i,t}^T \mathbf{E}^T r_{i,t}^*(p_{i,t}) \quad (23)$$

s.t.

$$\underline{r} \leq \mathbf{E}^T r_{i,t}^*(p_{i,t}) \leq \bar{r} \quad (24)$$

$$p_{i,t} \in \mathbf{P}_t \quad (25)$$

$$r_{i,t}^*(p_{i,t}) = \arg \max_{r_{i,t}} p_{i,t}^T \mathbf{E}^T r_{i,t} - \frac{1}{2} r_{i,t}^T \mathbf{C}_t r_{i,t} - \mathbf{b}_t^T r_{i,t} \quad \forall i \quad (26)$$

where ζ_t is the clearing prices for up- and down-reserve capacities determined through the electricity market clearing by system operator (SO) at time t ; \underline{r} , \bar{r} are the lower- and upper- bounds of up- and down-reserve capacities required by SO, respectively; $p_{i,t}$ is the price signal from the aggregator to consumer i for up- and down-reserve capacities at time t , which should lie in a price interval specified by (25); $r_{i,t}^*$ is the optimal response, which depends on $p_{i,t}$, from consumer i determined by the lower-level CRP' problem, i.e., (26) and (11)-(15). After receiving \underline{r} , \bar{r} required by SO, the goal of AP problem is to optimally decide its price signal $\{p_{i,t}\}$, so that its profit in (23) is maximized, while the total up- and down-reserve capacities gathered from consumers through the incentive meet the requirement of SO in (24).

For comparison, three methods are implemented to solve the AP problem using mpEC, PM and the proposed method, respectively. Note that when using the proposed method, the lower-level CRP' problem can be replaced by the piece-wise affine offer curve in (21) and (22). Finally, the bi-level problem is transformed equivalently into a mixed-integer quadratic program (MIQP), which can be solved efficiently by the existing commercial solvers such as Gurobi. The reserve requirement is set to be $\underline{r} = (0.8, 0.95)^T$ and $\bar{r} = (1.0, 1.1)^T$. For simplicity, $N=1$ is considered in the simulation. ζ_t is simplified as a constant vector arbitrarily, setting to be $(90.0, 90.0)^T$ when $p_{t,sup} \geq 20$, and $p_{t,down} \leq 100$.

Simulation results are compared in Table II. The results of mpEC can serve as a benchmark since the full model of consumers, i.e., cost functions, load profiles and other parameters in constraints, is collected by the aggregator and the exactness is guaranteed. Technically, the full model is represented by KKT conditions of the lower-level problem. However, the method cannot protect the privacy of consumers and thus may not be compatible with the electricity market. Instead of disclosing all the information of consumers, i.e., cost coefficients and load profiles, to the upper-level aggregator, a price-reserve offer curve that characterizes the response of consumers is submitted to the aggregator in both PM and the proposed MPEP, which preserves the privacy of consumers.

TABLE II
COMPARISON OF APPLYING THREE METHODS INTO BI-LEVEL AP PROBLEM

Method	$\{p_{t,down}, p_{t,sup}\}$ (\$)	$\{r_{t,down}, r_{t,sup}\}$ (p.u.)	Objective value (\$)	Computation time (ms)	Information collected from consumer	Privacy protection	Optimality
mpEC	{75.86, 20.0}	{0.95, 0.9}	76.44	190.2	Full model	No	Yes
PM(20)	{100.0, 20.0}	{1.1, 0.9}	52.00	400.0	Estimated offer curve	Yes	No
Proposed	{75.86, 20.0}	{0.95, 0.9}	76.44	272.9	Analytic offer curve	Yes	Yes

If using PM(20), the aggregator will misprice the down-reserve due to the error of the estimated offer curve, which unnecessarily costs itself 31.97% of the profit even in the simple case. As shown in Table I, tremendous extra computation time are required to reduce the error. Since the offer curve is obtained analytically, the price and reserve obtained by the proposed method are identical to the benchmark, while the privacy of the consumer is protected.

VI. CONCLUSION AND DISCUSSION

Based on the proposed MPEP, an efficient analytic method is presented to compute the price-reserve offer curve at the consumer level. An example of its application in the bi-level AP problem is also discussed. Numerical results show that the proposed method is both efficient and accurate compared with the existing methods, while the privacy of consumers is well preserved. Even in the simple illustrating example, the proposed method brings significant economic improvement due to the accuracy of the calculated offer curve compared with the existing methods that use stepwise constant offer curves. The improvement is due to a more accurate description of the optimal reserve using the proposed analytic method.

Temporal constraints can be readily considered if the variables are extended to incorporate all time slots in the problem of CRP-A. Also, the proposed MPEP does not rely on the specific formulation of CRP model. If other problems can be written in the form of CRP-A, which is a quadratic programming problem, MPEP is still applicable.

However, this research work is preliminary. Large-scale demand-side flexibility aggregation is left open, where efficient decomposition, e.g., based on the alternating direction method of multipliers, is worthy of further study. In the future, we will also investigate a more detailed load model of consumers [13], and complicated physical constraints of three-phase power flow [14]. Uncertainties of loads could be addressed via the deep neural network in [15], which warrants future efforts.

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