

# Mixed Aleatory-epistemic Uncertainty Modeling of Wind Power Forecast Errors in Operation Reliability Evaluation of Power Systems

Jinfeng Ding, Kaigui Xie, Bo Hu, Changzheng Shao, Tao Niu, Chunyan Li, and Congcong Pan

**Abstract**—As the share of wind power in power systems continues to increase, the limited predictability of wind power generation brings serious potential risks to power system reliability. Previous research works have generally described the uncertainty of wind power forecast errors (WPFEs) based on normal distribution or other standard distribution models, which only characterize the aleatory uncertainty. In fact, epistemic uncertainty in WPFEE modeling due to limited data and knowledge should also be addressed. This paper proposes a multi-source information fusion method (MSIFM) to quantify WPFEEs when considering both aleatory and epistemic uncertainties. An extended focal element (EFE) selection method based on the adequacy of historical data is developed to consider the characteristics of WPFEEs. Two supplementary expert information sources are modeled to improve the accuracy in the case of insufficient historical data. An operation reliability evaluation technique is also developed considering the proposed WPFEE model. Finally, a double-layer Monte Carlo simulation method is introduced to generate a time-series output of the wind power. The effectiveness and accuracy of the proposed MSIFM are demonstrated through simulation results.

**Index Terms**—Wind power forecast error (WPFEE), epistemic uncertainty, multi-source information fusion method (MSIFM), operation reliability, extended focal element (EFE), double-layer Monte Carlo simulation.

## I. INTRODUCTION

WIND power is increasingly contributing to electricity supplies worldwide because of its low environmental impact and negligible generation costs [1]. Although many environmental benefits can be obtained, the uncertainty from considerable penetration of wind power also poses great challenges to the reliable operation of power systems. Reliable

electricity delivery is a core value in the power industry. Effectively and accurately analyzing the impact of wind power uncertainty on the operation reliability of power systems can provide a theoretical basis for the effective use of wind power. Reference [2] quantifies both the economic and reliability effects of improved wind power forecast. Reference [3] incorporates wind power into the regional risk concept for operation reliability evaluation. Reference [4] proposes an improved importance sampling method to evaluate the reliability of composite power systems with wind energy integration. To measure the uncertainty of wind power output accurately, wind power forecast techniques have been rapidly developed [5]. If the forecast technique is sufficient, the forecast error will be similar to white noise and exhibit completely random behavior [6], [7]. However, it is widely understood that wind power cannot be forecasted at high accuracy at all times, and analyzing the uncertainty of wind power forecast errors (WPFEEs) can provide more useful information.

In recent years, researchers have performed extensive and thorough studies on how to apply forecast error information based on diverse perspectives and broaden its applications in different fields. The effects of WPFEEs on unit commitment, economic dispatch, and branch limit violations have been studied [8], [9]. Reference [10] proposes a data preprocessing method with error correction to improve the forecast accuracy of wind power generation. WPFEEs also affect the power balance of power systems and further affect the reliability of power systems. Reference [11] analyzes the effects of the standard deviation of forecast errors on reliability sensitivities.

In most previous studies, WPFEEs are often assumed to follow a normal distribution or the so-called  $\beta$  distribution [12]. However, some research works have shown that the probability density function (PDF) of WPFEEs varies with different forecast techniques and seasons. WPFEEs may not follow a common distribution form [7]. Therefore, some research works apply non-parametric methods to model WPFEEs. Kernel density estimation is a popular method for estimating data distribution without the prior assumption of datasets [13], [14]. However, these types of methods require sufficient historical data to achieve sufficient accuracy. The lower-upper bound estimation method is used to construct the forecast interval of wind power [15], [16]. Interval method (IM) cannot make full use of known information, and the uncertainty model has not been well quantified [17]. Thus, the obtained

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J. Ding, K. Xie, B. Hu, C. Shao (corresponding author), T. Niu, C. Li, and C. Pan are with the Power and Energy Reliability Research Center, State Key Laboratory of Power Transmission Equipment & System Security and New Technology, Chongqing University, Chongqing, 400044, China (e-mail: jinfengding@cqu.edu.cn; kaiguixie@vip.163.com; hboy8361@163.com; cshao@cqu.edu.cn; tao-niu@cqu.edu.cn; lcyqqu@cqu.edu.cn; pance@cqu.edu.cn).

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results are typically conservative.

The aforementioned methods either consider the selected probability distribution model to be perfectly accurate for modeling WPFEs or assume that the historical data are sufficient for parameter estimation. In reality, the two assumptions are not always satisfied, which leads to epistemic uncertainty [18], [19]. Epistemic uncertainty is another type of basic uncertainty that substantially differs from aleatory uncertainty [20]. Aleatory uncertainty stems from the variability of wind, whereas epistemic uncertainty results from incomplete knowledge of the rules of wind power.

Previous studies have seldom quantified the uncertainty of WPFEs with sufficient accuracy because only aleatory uncertainty has been considered. The inaccurate modeling of WPFEs may lead to an overly optimistic assessment of power system reliability and may pose potential risks to power system operations. By contrast, epistemic uncertainty is a type of uncertainty that can be reduced by acquiring additional knowledge. It is also critical to reduce epistemic uncertainty by applying as much known information as possible and using appropriate methods. Thus, quantifying epistemic uncertainty in WPFEE modeling is necessary, which has rarely been investigated to date.

To fill this research gap, this paper proposes a set of methods that can describe both the aleatory and epistemic uncertainties of WPFEs within the same framework based on evidence theory. Evidence theory is widely regarded as a promising mathematical tool for epistemic uncertainty analysis [21]. However, the traditional focal element method is generally not suitable for WPFEE modeling. Accordingly, the concept of an extended focal element (EFE) is introduced by considering the characteristics of wind power, and a method for selecting the EFE based on the sufficiency of historical data is proposed to improve the accuracies of the methods.

In addition, because expert information can be obtained from researchers and operators to assist in modeling, this paper constructs two types of expert information and proposes a multi-source information fusion method (MSIFM) to deal with situations of insufficient historical data. Discount factors are used to measure the credibility of multi-source information.

The proposed WPFEE model is incorporated into an operation reliability evaluation framework of power system to analyze the impact of high-penetration wind power on the reliability indices. Because of the need to simultaneously deal with two diverse uncertainties, this paper also proposes a double-layer Monte Carlo simulation (MCS) method for evaluating the operation reliability of power systems, where the outer and inner layers of the model handle aleatory and epistemic uncertainties, respectively.

The innovative contributions of the proposed method are summarized as follows:

1) To characterize the epistemic uncertainty caused by insufficient data or knowledge, a WPFEE modeling method is proposed and applied to the operation reliability evaluation of power systems.

2) Two types of expert information are constructed and combined with historical data through the proposed MSIFM to improve the accuracy of the model.

3) EFEs are proposed to reflect the characteristics of wind power more accurately, and the principle of choosing the number of EFEs is studied.

4) A double-layer MCS method is proposed that can deal with aleatory and epistemic uncertainties in the same framework.

The remainder of this paper is organized as follows. The uncertainty model of WPFEs is introduced in Section II. Section III proposes an operation reliability evaluation framework. Case studies are presented in Section IV. Section V presents conclusions derived from this paper.

## II. UNCERTAINTY MODEL OF WPFES

Wind power generation closely depends on natural factors and tends to suffer from the chaotic nature of weather systems. Therefore, traditional wind power forecasts cannot avoid errors. The WPFEE represents an uncertainty characterization between the forecast value and the true value. In this paper, the wind power output  $P^W$  is modeled as the sum of the forecast value  $F^W$  and WPFEE  $e^W$ :

$$P^W = F^W + e^W \quad (1)$$

Probability distribution fitting is a common method used to describe aleatory uncertainty. It is generally believed that WPFEs conform to a normal distribution. WPFEs can be sampled from a fitting curve following the parameter fitting of the historical data. Under these assumptions, the aleatory uncertainty of WPFEs can be accurately described as long as the sample size is sufficiently large. However, the normal distribution fitting (NDF) curve of WPFEs has a large deviation from actual situations, as shown in Fig. 1.

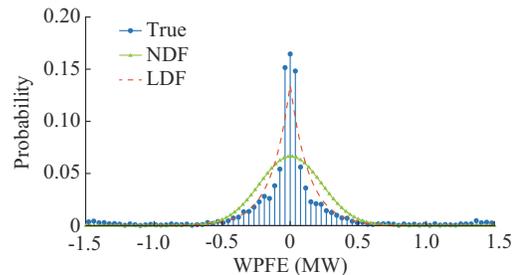


Fig. 1. NDF and LDF of WPFEEs.

In Fig. 1, the blue matchstick chart is the statistical data of true WPFEEs from a wind farm in Northwest China [7]. Some research works have assumed that WPFEEs conform to the Laplace distribution. It can be observed that the Laplace distribution fitting (LDF) model is sharper than the NDF model. However, the LDF model increases the probability of events with relatively small WPFEEs to a certain extent.

Two reasons can explain the differences between the distribution fitting model and the actual data. First, WPFEEs may not follow a specific distribution. Several research works have shown that describing WPFEEs with any general distribution form is difficult considering the diverse forecast methods and seasonal conditions [7]. Second, there is the possible lack of data for accurately estimating the parameters of the distribution model. These two factors lead to epistemic uncertainty. Epistemic uncertainty arises from imperfect

knowledge or ignorance, which is also referred to as subjective uncertainty, reducible uncertainty, or model form uncertainty. By contrast, aleatory uncertainty is an inherent variation associated with a parameter, physical system, or environment, and is also referred to as variability, stochastic uncertainty, or irreducible uncertainty [20]. Thus, obvious errors are inevitable in WPFEE modeling when considering only aleatory uncertainty. Subsequently, the accuracy of the operation reliability evaluation of power system can be negatively affected. As the penetration level of wind power continues to increase, considering both aleatory and epistemic uncertainties of WPFEEs is critical when evaluating the operation reliability of power system. To this end, this paper proposes a refined WPFEE model that captures both aleatory and epistemic uncertainties.

### A. Uncertainty Modeling Framework for WPFEEs

First, the fluctuation range of WPFEEs is divided into  $N$  intervals, and each interval  $[e_k^W, e_{k+1}^W]$  is denoted as a basic element  $E_k$ . The width  $d$  of each interval is equal such that:

$$e_k^W = e_1^W + (k-1)d \quad (2)$$

$$d = (e_{N+1}^W - e_1^W) / N \quad (3)$$

where  $k$  is the index of  $e_k^W$ ; and  $e_1^W$  and  $e_{N+1}^W$  are the lower and upper bounds of the WPFEEs, respectively.

Then, an identification frame  $\Theta$  is established to denote a set of all  $E_k$ :

$$\Theta = \{E_1, E_2, \dots, E_N\} \quad (4)$$

The power set of  $\Theta$  is denoted as  $2^\Theta$ . Any subset of  $\Theta$  can be denoted as an event  $A$  that belongs to  $2^\Theta$ :

$$A \in 2^\Theta: \{\emptyset\}, \{E_1\}, \{E_2\}, \dots, \{E_N\}, \{E_1, E_2\}, \{E_1, E_3\}, \dots, \{E_1, E_2, \dots, E_k\}, \{E_1, E_3, \dots, E_{k+1}\}, \dots, \Theta \quad (5)$$

A basic probability assignment (BPA) (also called a mass function  $m$ ) is introduced to allocate a certain probability to each event such that:

$$\sum_{A \in 2^\Theta} m(A) = 1 \quad (6)$$

$$m(A) \geq 0 \quad (7)$$

$$m(\emptyset) = 0 \quad (8)$$

Equation (7) indicates that the probability of an event cannot be negative. The empty set is meaningless for WPFEEs, and the probability of  $\emptyset$  is set to be zero. Any event  $A$  with  $m(A) > 0$  is called a focal element in traditional evidence theory. Some events  $A$  with  $m(A) = 0$  must still be considered because of the characteristics of WPFEEs and the lack of historical data. Otherwise, the WPFEE modeling will be inaccurate. Therefore, an EFE  $H$  is proposed in this paper, and  $H_1-H_4$  are four different  $H$ . An EFE can be any focal or basic element.

Traditionally, the probability of a WPFEE is determined by aleatory uncertainty. However, the probability cannot be accurately obtained due to insufficient data or knowledge. To characterize the epistemic uncertainty, extending the probability of WPFEE to a probability interval is more reasonable.

The belief function  $Bel(H_1)$  is defined as a measure of the lower bound probability, and the plausibility function  $Pl(H_1)$  is defined as a measure of the upper bound probability, which can be calculated as:

$$Bel(H_1) = \sum_{H_2 \subseteq H_1} m(H_2) \quad \forall H_1, H_2 \subseteq \Theta \quad (9)$$

$$Pl(H_1) = \sum_{H_2 \cap H_1 \neq \emptyset} m(H_2) \quad \forall H_1, H_2 \subseteq \Theta \quad (10)$$

The actual probability  $P(H)$  of EFE is determined by the belief and plausibility functions such that:

$$Bel(H) \leq P(H) \leq Pl(H) \quad (11)$$

The uncertainty model of WPFEEs is illustrated in Fig. 2.

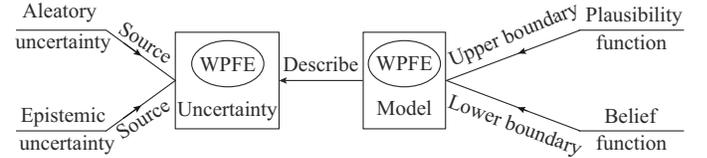


Fig. 2. Uncertainty model of WPFEEs.

As shown in Fig. 3, the degree of epistemic uncertainty for the observed EFE  $H$  can be represented by the width of the interval  $[Bel(H), Pl(H)]$ , which is related to the amount of known information. The number of EFEs can be appropriately increased when the amount of known information increases. Then, EFE  $H$  will be reduced to EFE  $H'$ , and the degree of epistemic uncertainty will also be reduced. The WPFEE may belong to any EFE because of its aleatory uncertainty. If the degree of epistemic uncertainty is reduced to zero because sufficient and accurate information is first obtained and then mastered, only aleatory uncertainty will remain in the model.  $Bel$  and  $Pl$  will converge to the probability distribution of the WPFEE.

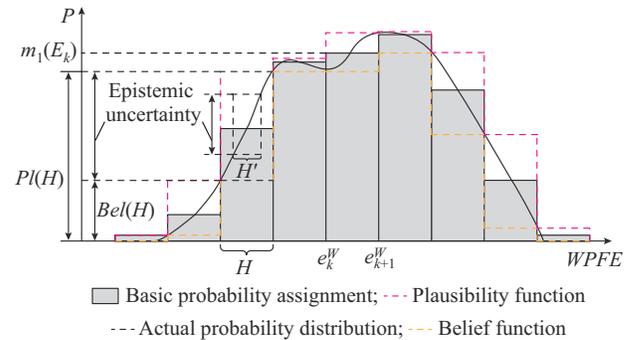


Fig. 3. Belief and plausibility functions.

The number of EFEs is crucial for ensuring the accuracy of the model and must be set reasonably according to the sufficiency of historical data.

### B. Construction of Multi-source Information

The frequency statistics method is used to obtain BPA  $m_0$  based on historical data. Each WPFEE value will only count towards the basic element that contains the value. Thus, the number of EFEs derived from historical data equals the number of basic elements. The procedure for the method is shown

in Algorithm 1, where  $N_{his}$  is the number of historical data.

**Algorithm 1:** frequency statistics method for calculating BPA  $m_0$  derived from historical data

**Input:** historical data and number of basic elements  $N$

**Output:** BPA  $m_0$  derived from historical data

- 1: **initialize:** set  $m_0(E_k)=0$ , where  $k=1, 2, \dots, N$
- 2: **for**  $k=1, 2, \dots, N$  **do**
- 3:     **if** a historical datum belongs to  $E_k$  **then**
- 4:          $m_0(E_k)=m_0(E_k)+\frac{1}{N_{his}}$
- 5:     **end if**
- 6: **end for**

Although we have obtained the original BPA derived from historical data, full belief in historical data may lead to incorrect results because insufficient historical data are a major source of epistemic uncertainty. The sufficiency of historical data can be defined by the Kullback-Leibler divergence (KLD) [22], which measures the difference between two statistical distributions. The KLD for a historical dataset can be evaluated as:

$$KLD = \sum_{k=1}^N \left[ \hat{m}_0(E_k) \lg \hat{m}_0(E_k) - \hat{m}_0(E_k) \lg m_0(E_k) \right] \quad (12)$$

where  $\hat{m}_0$  is the BPA of the complete dataset.

Historical data are insufficient if the KLD is greater than 0.1, whereas historical data are sufficient if the KLD is less than 0.05.

To adjust the credibility of historical data, a discount factor  $\alpha$  is used to preprocess the original BPA.

$$m_1(H) = (1 - \alpha) m_0(H) \quad \forall H \subseteq \Theta \quad (13)$$

where  $m_1$  is the preprocessed BPA derived from historical data; and  $1 - \alpha$  denotes the credibility of historical data.

Epistemic uncertainty is significant if historical data are insufficient. Therefore, the discount factor should be large to describe the low credibility of historical data in this situation. By contrast, epistemic uncertainty is mild if historical data are sufficient. The discount factor should be small to ensure that the epistemic uncertainty is accurately described within a reasonable range. The detailed selection principles of the discount factor are presented in Table I.

TABLE I  
SELECTION PRINCIPLES OF DISCOUNT FACTOR

KLD	Discount factor
$\geq 0.1$	$\geq 0.2$
$0.05 - 0.1$	$0.05 - 0.2$
$\leq 0.05$	$\leq 0.05$

However,  $m_1$  does not meet (6) because of the remaining unassigned probability determined by the discount factor. This part of the probability represents epistemic uncertainty. The probability should be assigned to the interval  $[\underline{e}^W, \bar{e}^W]$  to which all historical data belong, where  $\underline{e}^W$  and  $\bar{e}^W$  are the minimum and maximum values of the historical data, respectively. Note that BPA is built on  $2^\Theta$ . The calculation for BPA must be based on the EFE. Therefore, finding the minimum

EFE  $\{E_p, \dots, E_q\}$  is necessary such that  $[\underline{e}^W, \bar{e}^W] \subseteq [e_p^W, e_{q+1}^W]$ , where  $p$  and  $q$  are indices of the basic elements, which can be calculated as:

$$p = \arg \max_{e_k^W \leq \underline{e}^W, k=1, 2, \dots, N} k \quad (14)$$

$$q = \arg \min_{e_{k+1}^W \geq \bar{e}^W, k=1, 2, \dots, N} k \quad (15)$$

$$1 \leq p \leq q \leq N \quad (16)$$

The remaining probability is then assigned to  $\{E_p, \dots, E_q\}$  as:

$$m_1(H) = \alpha \quad H = \{E_p, \dots, E_q\} \quad (17)$$

In addition to the historical data of WPFEs, expert information is used to improve the accuracy of the WPF model. For example, previous research works have shown that the PDF of a WPF is conditional on its forecast value, where the forecast error has high bias and low variance when the forecast value is close to the upper and lower limits, and vice versa [23]. Operators can also provide effective experience or suggestions. Combining these valuable research findings, this paper constructs two types of expert information for calculating the BPAs of WPFEs: expert informations I and II.

Expert information I is modeled to describe the relationship between the WPF and predicted wind power value. Coefficient  $\beta$  is also introduced to determine the thresholds of the forecast values close to the upper or lower limits. The width of the critical region  $R_1$  is defined as:

$$R_1 = \beta P_e^W \quad (18)$$

where  $P_e^W$  is the rated power of the wind turbine.

The upper and lower critical regions are set to be  $[P_e - R_1, P_e]$  and  $[0, R_1]$ , respectively. Expert information I will cause the forecast value in the critical region to be closer to the boundary after modification by the WPF.

If the forecast value pertains to the upper critical region, the probability of the forecast error in the interval  $[0, R_1]$  is higher. BPA  $m_2$  derived from expert information I is formulated as:

$$m_2(H) = \gamma + (1 - \gamma) m_0(H) \quad H = \{E_p, \dots, E_q\} \quad (19)$$

$$m_2(H) = (1 - \gamma) m_0(H) \quad \forall H \subseteq \Theta, H \neq \{E_p, \dots, E_q\} \quad (20)$$

where  $\gamma$  is the credibility of expert information I; and  $\{E_p, \dots, E_q\}$  is the minimum EFE such that  $[0, R_1] \subseteq [e_p^W, e_{q+1}^W]$ .

If the forecast value belongs to the lower critical region, the probability of the forecast error in the interval  $[-R_1, 0]$  is higher. Similar methods have been used to obtain BPA for this situation.

Expert information II is modeled to represent the impact of extreme historical data. The lack of historical data will lead to narrowing the upper and lower limits of WPFs. Therefore, appropriately widening the boundary of WPFs based on the extreme value and credibility of historical data is necessary. The boundary value  $R_2$  is calculated as:

$$R_2 = \max(|\underline{e}^W|, |\bar{e}^W|) / (1 - \alpha) \quad (21)$$

In addition, it is known from operation experience that any possible WPFE in  $\Theta$  cannot be ignored. Thus,  $\Theta$  also must be assigned a probability based on the credibility of historical data. The BPA  $m_3$  derived from expert information II is given as:

$$m_3(H) = \alpha/2 + (1-\alpha)m_0(H) \quad H = \{E_p, \dots, E_q\} \quad (22)$$

$$m_3(H) = \alpha/2 + (1-\alpha)m_0(H) \quad H = \Theta \quad (23)$$

$$m_3(H) = (1-\alpha)m_0(H) \quad \forall A \subset \Theta, H \neq \{E_p, \dots, E_q\} \quad (24)$$

### C. Fusion of Multi-source Information

Notably, the fusion of multi-source information may help improve the accuracy. Moreover, the conflict among multi-source information may have a negative effect on the accuracy of the fusion. To avoid adverse effects and prevent the introduction of greater uncertainties after the fusion, based on evidence theory [24], this paper proposes an improved fusion method that is suitable for the aforementioned WPFE model.

$$q(H_4) = \sum_{H_1, H_2, H_3 \in \Theta, H_1 \cap H_2 \cap H_3 = H_4} m_1(H_1)m_2(H_2)m_3(H_3) \quad (25)$$

where  $q(H_4)$  is a probability assignment function and the fused BPA  $m_4$  is given as:

$$m_4(H) = 0 \quad H = \emptyset \quad (26)$$

$$m_4(H) = q(H) + wq(H)q(\emptyset) \quad H \neq \emptyset, \Theta \quad (27)$$

$$m_4(H) = q(H) + wq(H)q(\emptyset) + (1+wq(\emptyset) - w)q(\emptyset) \quad (28)$$

$$w = \frac{1-\alpha}{1-q(\emptyset) - q(\Theta)} \quad (29)$$

where  $w$  is the scale factor used for normalization, which is related to the credibility of historical data. The preprocessed historical data and expert information are fused to obtain BPA  $m_4$ .

The uncertainty model of WPFES is built based on the idea of the cumulative distribution function (CDF). The probability  $P_{WPFE}$  of the basic elements satisfies the following constraints:

$$\begin{aligned} 0 \leq Bel(\{E_1, E_2, \dots, E_{k-1}\}) \leq P_{WPFE}(E_k) \leq \\ Pl(\{E_1, E_2, \dots, E_{k-1}\}) \leq 1 \end{aligned} \quad (30)$$

where  $\{E_1, E_2, \dots, E_{k-1}\}$  is used to calculate  $Bel$  and  $Pl$ , as any WPFE value in  $E_k$  must be greater than or equal to the maximum WPFE value in  $\{E_1, E_2, \dots, E_{k-1}\}$ . The calculations for  $Bel$  and  $Pl$  are based on  $m_4$ .

To accurately describe the characteristics of WPFES during different periods of a single day, the proposed method is used to model WPFES for each hour of the day.

## III. OPERATION RELIABILITY EVALUATION FRAMEWORK

### A. Generation of Random Wind Power Output

WPFES have both aleatory and epistemic uncertainties.

Therefore, the traditional sampling method is not suitable for the proposed WPFE model. In this paper, a double-layer MCS method is used to generate a random output sequence of wind power.

First, the outer-layer MCS is used to randomly choose the EFE of the WPFES. This step deals with the aleatory uncertainty of WPFES. Extreme WPFES have low probabilities, but these may cause severe reliability accidents. To accurately and quickly assess the risks associated with these low-probability and high-impact WPFES, the outer-layer MCS adopts Latin hypercube sampling (LHS), which is a variance reduction technique based on stratification [25]. A random number  $u_1$  ranging from 0 to 1 is selected in this step. A corresponding relationship exists between  $u_1$  and the EFE. The EFE will be found if  $u_1$  is given as:

$$e^W(u_1) = \begin{cases} [e_1^W, e_2^W] & u_1 \in P_{WPFE}(E_1) \\ \vdots & \\ [e_r^W, e_{r+1}^W] & u_1 \in P_{WPFE}(E_r) \\ \vdots & \\ [e_s^W, e_{s+1}^W] & u_1 \in P_{WPFE}(E_s) \\ \vdots & \\ [e_N^W, e_{N+1}^W] & u_1 \in P_{WPFE}(E_N) \end{cases} \quad (31)$$

where  $r$ ,  $s$ , and  $N$  are consecutive indices of basic elements that yield  $1 \leq r \leq s \leq N$ .

Several  $P_{WPFE}$  may overlap because of the epistemic uncertainty of WPFES. Therefore,  $u_1$  may correspond to multiple consecutive EFEs. Then, the left and right boundaries of  $E_r$  and  $E_s$  generate the sampled interval  $[e_r^W, e_{s+1}^W]$  of the outer-layer MCS, respectively. The width of  $[e_r^W, e_{s+1}^W]$  is determined by epistemic uncertainty.

The inner-layer MCS is used for randomly choosing the sampled value of the WPFES. This step deals with the epistemic uncertainty of WPFES. No significant difference exists in the probability of WPFES in the interval  $[e_r^W, e_{s+1}^W]$  if the number of EFEs is selected appropriately. Therefore, the inner-layer MCS is used to sample the interval that follows a uniform distribution. The random number  $u_2$  selected in this step determines the sampled value  $e^W$  of the WPFES from  $[e_r^W, e_{s+1}^W]$ . An example of a double-layer MCS is shown in Fig. 4.

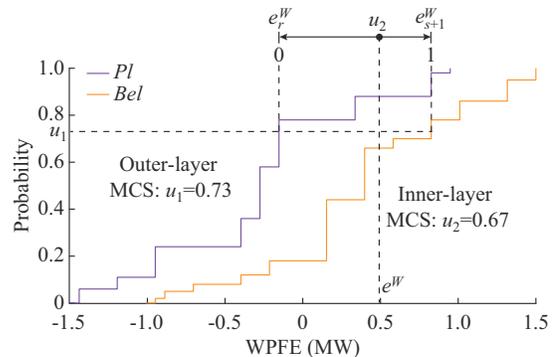


Fig. 4. Double-layer MCS.

The sampled value of the wind power output can be obtained by summing the forecast and WPFE sampled values using (1).

The double-layer MCS method is repeated to obtain the wind power output sequence for a given period.

### B. Component Failure Simulation

The state duration sampling method is used to generate the state sequence of the component [26]. Suppose that each component has two states (up and down), and the component state duration is exponentially distributed. First, all components are initially in the up state. A uniformly distributed random number  $\gamma_j$  between  $[0,1]$  is sampled to determine the duration of the  $j^{\text{th}}$  component residing in its present state. The state duration of the  $j^{\text{th}}$  component can be calculated as:

$$\xi_j = -\frac{1}{\lambda_j} \ln \gamma_j \quad (32)$$

where  $\lambda_j$  is the transition rate of the  $j^{\text{th}}$  component. If the  $j^{\text{th}}$  component is in the up state,  $\lambda_j$  is the failure rate; if the  $j^{\text{th}}$  component is in the down state,  $\lambda_j$  is the repair rate.

Equation (32) is repeated until the total simulation time is equal to or greater than the given time span to construct the chronological state transition process of each component. Then, the state sequence of each component is established.

### C. System State Analysis

The simulated operation of the system is also assessed. The wind power output and operation state of each component are obtained from the wind power output sequence and component state sequence, respectively, and are used as the known values of the system state analysis at any time  $t$ .

The following optimization model for minimizing the load curtailment is used to reschedule generation outputs to maintain the generation-demand balance, alleviate line overloads and avoid load curtailment, if possible, or minimize total load curtailment if unavoidable.

$$\min E \left( \sum_t f(\dot{x}) \right) \quad (33)$$

$$\phi(\dot{x}) = 0 \quad (34)$$

$$\varphi(\dot{x}) \leq 0 \quad (35)$$

$$g \in G, l \in L, i \in B, t \in T \quad (36)$$

where  $T$  is the evaluation period which is 24 hour in this paper.

In this model,  $\dot{x}$  indicates continuous variables:

$$\dot{x} = \left\{ P_{g,t,h}^G, P_{l,t,h}^L, P_{i,t,h}^{Loss}, \delta_{i,t,h} \mid \forall g, \forall l, \forall i, \forall t \right\} \quad (37)$$

where  $h$  is the index of a given evaluation day;  $P_{g,t}^G$  and  $P_{l,t}^L$  are the active power output of unit  $g$  and power flow of line  $l$  at time  $t$ , respectively;  $P_{i,t}^{Loss}$  is the load curtailment of bus  $i$  at time  $t$ ; and  $\delta_{i,t}$  is the phase angle of bus  $i$  at time  $t$ .

Constraints (34)-(36) include the power balance for each bus, power flow limits of every transmission line, limits of the power generators, limits of ramp up and down, limits of load curtailment, phase angles, and availability of components.

The system reliability indices  $LOLP$  and  $EENS$  are uti-

lized to evaluate system reliability [27].

$LOLP$  is defined as the loss of load probability, which can be expressed as:

$$LOLP = \frac{1}{TD} \sum_{t \in T, h \in D} I_{t,h} \quad (38)$$

$$I_{t,h} = \begin{cases} 1 & \sum_{i \in B} P_{i,t,h}^{loss} > 0 \\ 0 & \sum_{i \in B} P_{i,t,h}^{loss} = 0 \end{cases} \quad (39)$$

where  $D$  is the total number of evaluation days.

$EENS$  is defined as the expected energy which is not supplied, which can be expressed as:

$$EENS = \frac{1}{TD} \sum_{i \in B, t \in T, h \in D} P_{i,t,h}^{loss} \quad (40)$$

The flow of this process is shown in Fig. 5.

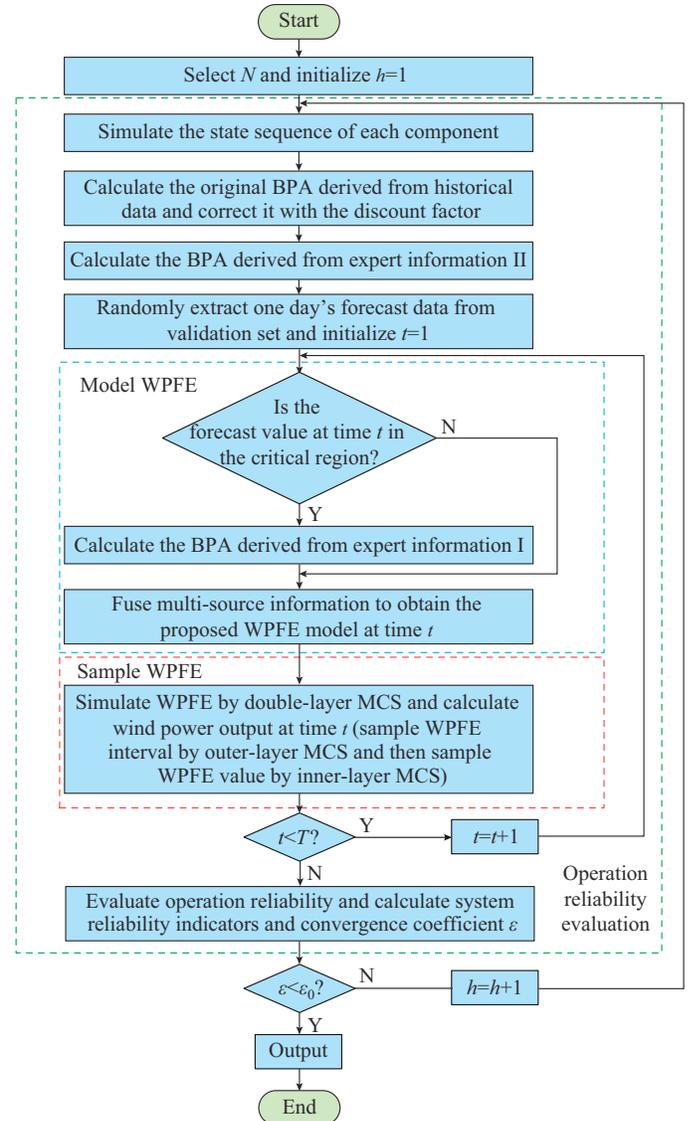


Fig. 5. Flow of operation reliability evaluation of power systems.

## IV. CASE STUDY

The proposed methods are applied to a modified Roy

Billinton Test System (RBTS) [28]. The peak load of the test system increase to 240 MW. A 300 MW wind farm with 200 wind turbines with a rated power of 1.5 MW is added to bus 1. The actual and forecast data of the wind power outputs are obtained from a wind farm in Northwest China. The WPFE data are calculated based on (1).

#### A. WPFE Model Based on Historical Data

The WPFEs are first modeled with complete historical data based on the proposed method. The reliability evaluation results based on the LDF model tend to be overoptimistic because the LDF model increases the probability of events with relatively small WPFEs. Thus, for comparison, the NDF method is used as a typical distribution fitting method.

Figure 6 demonstrates the CDFs of WPFEs modeled with complete historical data, where the numbers of EFEs in the four subgraphs are selected as 5, 10, 20, and 50, respectively. The blue (true) curve derives from the original WPFE data and is used as the reference. The fitting curve of the normal distribution is roughly close to the reference curve, but it is too flat if the absolute value of the WPFE is small. The proposed method also envelops the reference curve well. At every point, the true WPFE value is always between *Bel* and *Pl*. For any possible value of WPFE, the difference between *Bel* and *Pl* represents the degree of epistemic uncertainty. For illustration purposes, only two boxes in each subgraph are marked with epistemic uncertainty. The greater the number of EFEs selected, the smaller the difference between *Bel* and *Pl*, which means that the degree of epistemic uncertainty is less.

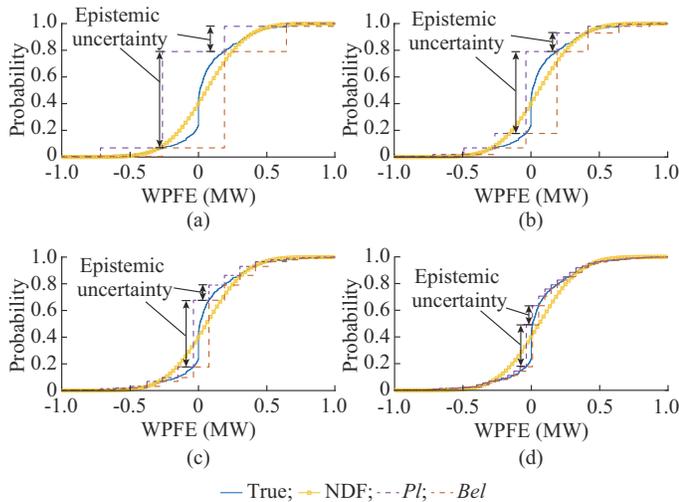


Fig. 6. CDFs of WPFEs modeled with complete historical data by different numbers of EFEs. (a) 5 EFEs. (b) 10 EFEs. (c) 20 EFEs. (d) 50 EFEs.

The distance between *Bel* and *Pl* further decreases to zero when the number of EFEs is sufficiently large. In other words, epistemic uncertainty is eliminated if sufficient historical data exist. Note that the situation in Fig. 6 appears because all historical data are used to construct *Bel* and *Pl*.

However, historical data obtained in actual situations are often limited. The WPFE data are divided into two parts. The first part is used to generate the WPFE model using the proposed method, and the second part is used as a validation

dataset. In our paper, the discount factor is set to be 0.1. As shown in Fig. 7, the number of EFEs is selected to be higher (e.g., 50) when the historical data are insufficient. The actual value of some WPFEs is completely out of the range of *Bel* and *Pl*. When the number of EFEs reduces to 10, *Bel* and *Pl* could cover all possible values of the WPFEs. However, the epistemic uncertainty of WPFEs also increases. Therefore, selecting an appropriate number of EFEs to model the WPFEs is necessary so that the proposed model could cover possible scenarios without causing excessive epistemic uncertainty.

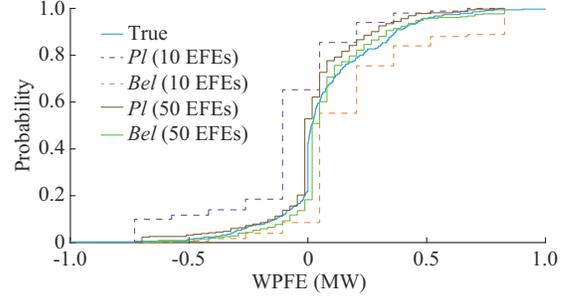


Fig. 7. CDFs of WPFEs modeled with incomplete historical data.

#### B. Fusion of Multi-source Information

The threshold coefficient  $\beta$  is set to be 0.1. The mean and variance of the true historical data are calculated and used as reference values for comparison. The calculation results are presented in the first two rows of Table II.

TABLE II  
MEAN AND VARIANCE OF DIFFERENT METHODS

Modeling method	Mean or variance	Lower critical region	Middle region	Upper critical region
True for reference	Mean	0.039	-0.029	-0.105
	Variance	0.022	0.079	0.048
NDF	Mean	0.029	-0.024	-0.087
	Variance	0.062	0.124	0.098
NFM	Mean	0.031	-0.028	-0.094
	Variance	0.058	0.114	0.083
MSIFM	Mean	0.036	-0.029	-0.101
	Variance	0.025	0.085	0.052

The mean and variance of the collected data conform to the rules found in [23]. The error of the forecast value in the critical region has a higher deviation and lower variance. This verifies the necessity of introducing expert information I.

The method based solely on historical data does not perform information fusion, and therefore it can also be called a non-fusion method (NFM). To compare the effects of the fusion, the double-layer MCS is used to sample the two WPFE models obtained by NFM and MSIFM, respectively. The comparison results between these two methods and the traditional NDF method are presented in Table II.

First, the errors of the forecast values in the middle region are compared. Little difference is observed in the mean val-

ues of the WPFEs simulated through the three methods. The variance values of the WPFEs simulated by NDF and NFM are relatively high, meaning that the simulated WPFEs are relatively scattered. The characteristic whereby the WPF is concentrated near the zero value is not previously described accurately. Second, the errors of the forecast values in the critical region are compared. The mean and variance values of the WPFEs simulated by NDF are significantly different from the reference values. Therefore, the risk caused by some extreme WPFEs may not be measured correctly, which is not conducive to the operation reliability evaluation. By contrast, NFM performs better than NDF in terms of WPF modeling. When the expert information is introduced to modify the model of the WPFEs, the mean and variance values of the WPFEs simulated by MSIFM are very close to the reference values in both the middle and critical regions.

The CDFs of the WPFEs modeled through different methods are compared in Fig. 8. The true WPF data are also plotted in this figure as a reference.

The NDF could not adequately describe the characteristics of the WPFEs. The curve of the NDF is relatively flat as compared with the true curve, indicating a large error. For the proposed model, the curve simulated by the double-layer MCS is roughly the middle line between *Bel* and *Pl*. Thus, even though NFM normally considers the credibility of historical data and correctly describes the epistemic uncertainty, the simulation results of NFM are still not sufficiently accurate. After expert information correction is conducted, the MSIFM reaches a very high accuracy.

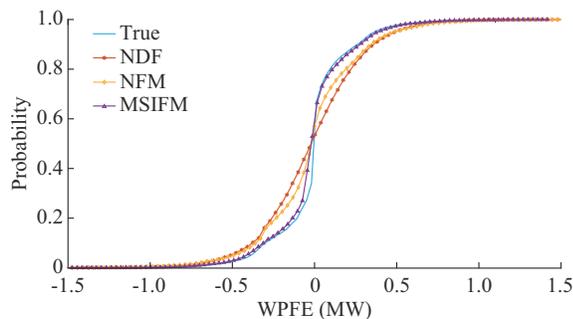


Fig. 8. CDFs of WPFEs modeled by different methods.

### C. Operation Reliability Evaluation

Results of operation reliability evaluation through different methods are presented in Table III. The references *LOLP* and *EENS* calculated using the true WPF data of the validation set are 0.0235 and 0.6380, respectively.

TABLE III  
RESULTS OF OPERATION RELIABILITY EVALUATION THROUGH DIFFERENT METHODS

Method	<i>LOLP</i>		<i>EENS</i>	
	Value	Error (%)	Value (MWh/day)	Error (%)
IM	0.0276	17.45	0.7445	16.70
NDF	0.0259	10.21	0.6976	9.35
LDF	0.0205	-12.77	0.5508	-13.67
MSIFM	0.0239	1.70	0.6423	0.67

Through accurate WPF modeling, the proposed method greatly improves the accuracy of the results of the operation reliability evaluation.

A historical dataset with 300 sets of daily data is considered. Under these circumstances, the KLD is 0.0488, and the discount factor is recommended to be 0.05. Two cases are then modeled in which the discount factors are set to be 0.2 and 0.05, respectively. The results of the operation reliability evaluation are presented in Table IV.

TABLE IV  
OPERATION RELIABILITY EVALUATION RESULTS WITH DIFFERENT DISCOUNT FACTORS

Discount factor	<i>EENS</i> (MWh/day)					Average error (%)
	Dataset 1	Dataset 2	Dataset 3	Dataset 4	Dataset 5	
0.20	0.6718	0.7197	0.6969	0.7259	0.6972	6.43
0.05	0.6401	0.6885	0.6769	0.6568	0.6672	2.79

As shown in Table IV, choosing a discount factor of 0.05 produces more precise reliability evaluation results. Numerical simulations demonstrate the effectiveness of the selection principle for the discount factor.

The *EENS* evaluation results of different numbers of days of known historical data are shown in Fig. 9.

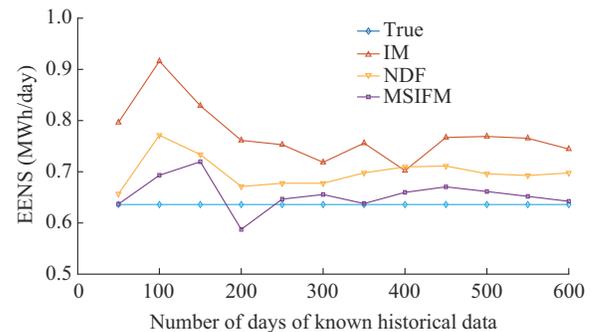


Fig. 9. *EENS* evaluation results of different numbers of days of known historical data.

These three methods gradually approach the convergence as the amount of historical day data increases. The results of IM are the most conservative. In general, this means that the cost is the highest if the system reliability needs to be improved. The results of NDF converge to a value with a greater error even when the amount of historical day data is sufficiently large. MSIFM achieves better performance than the traditional methods regardless of the amount of historical day data due to the appropriate number of EFes being determined. The relationship between the change in the number of EFes or the amount of historical day data and the *EENS* error is shown in Fig. 10(a). The error in the evaluation results decreases as the amount of historical day data increases. A clearer changing trend could be observed in the cumulative average error, as shown in Fig. 10(b).

The number of EFes is first selected to be 5, which represents the largest epistemic uncertainty. The results converge slowly and show the largest deviation when the amount of historical day data is sufficiently large. Then, the number of

EFEs is selected to be 50, which represents the smallest epistemic uncertainty. However, the results do not show a convergence trend. In fact, the known historical day data are not completely credible, and the estimation of epistemic uncertainty is insufficient. This situation is similar to that reported in Fig. 7, which leads to an incorrect trend. It could also be understood that the amount of historical day data still does not match this high number of EFEs. Finally, the numbers of EFEs are selected to be 10 and 20. Both results have high convergence accuracy when the amount of historical day data is sufficient. However, the results with 20 EFEs have larger errors when the amount of historical day data is small. Therefore, the cumulative average error with 20 EFEs is higher.

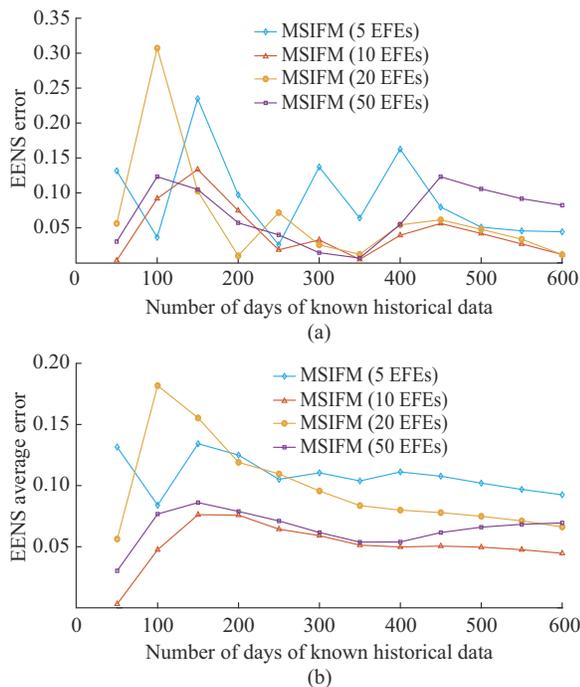


Fig. 10. EENS evaluation results with different numbers of EFEs. (a) Relative errors of EENS. (b) Cumulative average errors of EENS.

In short, the number of EFEs should be appropriately chosen to be in agreement with the amount of historical day data. The fundamental principle for determining the number of EFEs is that the number of EFEs must increase as the amount of historical day data increases. More specifically, the recommended number of EFEs is 5-10 if the amount of historical day data is small (when KLD in (12) is larger than 0.1), and should be 10-50 if the historical day data are relatively sufficient (when KLD in (12) is less than 0.05).

## V. CONCLUSION

This paper proposes a WPFE modeling method that considers epistemic uncertainty caused by insufficient data or knowledge. To improve the accuracy of the model, the MSIFM is proposed, and the number of EFEs is appropriately chosen based on the proposed method. The double-layer MCS is used to simulate both the aleatory and epistemic uncertainties of WPFEs in the same framework to evaluate the

operation reliability of power systems.

Simulation results demonstrate that accurate WPFE modeling is critical for operation reliability of power systems. The uncertainty characteristics of WPFEs described by the NDF method are proven to be not very precise. Combining the proposed WPFE model and double-layer MCS could obtain accurate evaluation results of operation reliability. In addition, the number of EFEs must be selected based on the amount of historical day data. More specifically, to improve the accuracy of the evaluation results, the number of EFEs must increase as the amount of historical day data increases.

This paper focuses on WPFE modeling, where WPFEs are mainly caused by the inherent variability associated with wind speed and insufficient historical data or knowledge necessary to precisely characterize it. Thus, the correlation between the WPFEs of different wind turbines is not considered. Nevertheless, the correlation of the WPFEs of different wind turbines can in fact be considered using the proposed method. The correlation information can be regarded as an independent information source and modeled as a type of expert information. In addition, the proposed method can still be used to characterize epistemic uncertainty caused by the unknown correlation of the forecast errors of multiple wind turbines.

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**Jinfeng Ding** received the B.S. degree in electrical engineering from Chongqing University, Chongqing, China, in 2019. He is currently pursuing the M.S. degree in electrical engineering from Chongqing University. His research interests include power system reliability evaluation, renewable energy, and power system optimization.

**Kaigui Xie** received the Ph.D. degree in power systems and automation in 2001 from Chongqing University, Chongqing, China, where he is currently a Full Professor with the School of Electrical Engineering. He is an Editor of IEEE Transactions on Power Systems and an Associate Editor for IET Proceedings-Generation, Transmission and Distribution. His research interests include power system reliability, planning, and analysis.

**Bo Hu** received the Ph.D. degree in electrical engineering from Chongqing University, Chongqing, China, in 2010, where he is currently a Professor with the School of Electrical Engineering from Chongqing University. His research interests include power system reliability and parallel computing techniques in power systems.

**Changzheng Shao** received the Ph.D. degree in electrical engineering from Zhejiang University, Hangzhou, China, in 2020. He is currently a Postdoctor in Chongqing University, Chongqing, China. His research interests include operation optimization and reliability evaluation of integrated energy systems.

**Tao Niu** received the B.S. and Ph.D. degrees in electrical engineering from Tsinghua University, Beijing, China, in 2014 and 2019, respectively. He is currently an Assistant Professor with Chongqing University, Chongqing, China. His research interests include voltage security region, automatic reactive power voltage control, renewable generation integration, and reactive power analysis of hybrid AC/DC systems.

**Chunyan Li** received the Ph.D. degree in electrical engineering in 2008 from Chongqing University, Chongqing, China, where she is currently an Associate Professor with the School of Electrical Engineering. Her research interests include power system analysis, renewable energy consumption, and smart grid dispatching.

**Congcong Pan** received the B.S. degree in electrical engineering from Northeast Electric Power University, Jilin, China, in 2019. He is currently pursuing the Ph.D. degree in the School of Electrical Engineering at Chongqing University, Chongqing, China. His research interests include supply-demand balance and risk modeling in power systems.