Optimal Offering of Energy Storage in UK Day-ahead Energy and Frequency Response Markets

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***Abstract*—The offering strategy of energy storage in energy and frequency response (FR) markets needs to account for country-specific market regulations around FR products as well as FR utilization factors, which are highly uncertain. To this end, a novel optimal offering model is proposed for stand-alone price-taking storage participants, which accounts for recent FR market design developments in the UK, namely the trade of FR products in time blocks, and the mutual exclusivity among the multiple FR products. The model consists of a day-ahead stage, devising optimal offers under uncertainty, and a real-time stage, representing the storage operation after uncertainty is materialized. Furthermore, a concrete methodological framework is developed for comparing different approaches around the anticipation of uncertain FR utilization factors (deterministic one based on expected values, deterministic one based on worst-case values, stochastic one, and robust one), by providing four alternative formulations for the real-time stage of the proposed offering model, and carrying out an out-of-sample validation of the four model instances. Finally, case studies employing real data from UK energy and FR markets compare these four instances against achieved profits, FR delivery violations, and computational scalability.**

***Index Terms*—Energy markets, energy storage, frequency response, optimal offering, robust optimization, stochastic programming.**

Nomenclature

A. Indices and Sets

e, o	Indices of time blocks
E	Set of time blocks
r, R	Index and set of out-of-sample scenarios
s, S	Index and set of price scenarios
t, i	Indices of time periods
T	Set of time periods
T_e	Set of time periods included in time block e
w, W	Index and set of frequency response (FR) utilization scenarios
x, X	Index and set of FR products

B. Parameters

$\alpha_{t,w}^{x,U}$, $\alpha_{t,w}^{x,D}$	Utilization factors of upward and downward provisions of FR product x during time period t in scenario w (MWh/MW)
$\lambda_{t,s}^{DA}$	Day-ahead energy price during time period t in scenario s (£/MWh)
$\lambda_{e,s}^{x,U}$, $\lambda_{e,s}^{x,D}$	Prices of upward and downward provisions of FR product x in time block e and scenario s (£/MW/h)
π_s	Probability of price scenario s
π_w	Probability of FR utilization scenario w
Δt	Duration of time period t (hour)
Δe	Duration of time block e (hour)
n^{ch}, n^{dch}	Charging and discharging efficiencies of storage
$\overline{P}$	Upper power limit of (dis)charging of storage (MW)
$\overline{SOC}$	Upper limit of state of charge of storage (MWh)
$\underline{SOC}$	Lower limit of state of charge of storage (MWh)
SOC_0	Initial state of charge of storage (MWh)
$UT_e^{x,U}$, $UT_e^{x,D}$	Uncertainty budgets limiting the worst-case aggregated utilization factor of upward and downward provisions of FR product x over time block e (hour)

C. Variables

$\alpha_{i,t}^{x,U}$, $\alpha_{i,t}^{x,D}$	Utilization factors of upward and downward provisions of FR product x during time period i considered in inner problem of time period t (MWh/MW)
$l_{t,r}^{+}$, $l_{t,r}^{-}$	Over-delivery and under-delivery violations associated with FR delivery during time period t in out-of-sample scenario r (MW)
$p_t^{DA,S}$, $p_t^{DA,B}$	Day-ahead energy selling and buying offers during time period t (MW)
$p_e^{x,U}$, $p_e^{x,D}$	Offers for upward and downward provisions of FR product x in time block e (MW)
$p_{t,w}^{ch}$, $p_{t,w}^{dch}$	Charging and discharging power during time period t in scenario w (MW)

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p_t^{ch}, p_t^{dch} Charging and discharging power during time period t (MW)

$SOC_t^{\max}$, $SOC_t^{\min}$ The maximum and minimum possible states of charge of storage during time period t (MWh)

$SOC_{t,w}$ State of charge during time period t in scenario w (MWh)

SOC_t State of charge during time period t (MWh)

u_e^x Binary variable expressing whether an offer for FR product x in time block e is submitted ($u_e^x=1$) or not ($u_e^x=0$)

$z_{t,w}^{ch}$ Binary variable expressing whether the storage charges ($z_{t,w}^{ch}=1$) or discharges ($z_{t,w}^{ch}=0$) during time period t in scenario w

I. Introduction

The unprecedented penetration of renewable energy sources (RESs) in modern power systems, as the major avenue towards the decarbonization of the energy industry, introduces novel challenges, mainly associated with the stochasticity of RESs. These challenges urgently call for a significant increase of the flexibility of power systems [1]. Energy storage constitutes a fundamental component of this new paradigm, due to its intrinsic ability to act as both generation and demand, and balance accordingly the mismatches emerging from the stochasticity of RESs [2].

Considering the deregulation of electricity markets across the world, the realization of the flexibility value of energy storage needs to be realized through its appropriate participation in electricity markets. Specifically, energy storage owners constitute self-interested participants, which explore various market opportunities with the aim of maximizing their profit. Such opportunities may refer to both national transmission and local distribution levels [3]. Focusing on the former, there are two crucial opportunities for energy storage in wholesale electricity markets. The first one is associated with arbitrage in energy markets, while the second one refers to the provision of frequency response (FR), the value of which is constantly increasing due to the increasing penetration of RESs. Nevertheless, these two opportunities cannot be explored in silos, since the upward/downward flexibility provided by energy storage for FR is specified with respect to its baseline energy levels.

Focusing on the UK market, some rapid developments are observed with respect to the market design for the provision of various FR products [4], [5]. The first of these is the trading of such products on a day-ahead basis, allowing for participants to co-optimize their offers across energy and FR markets that are cleared with a common time horizon. Another development refers to the move from trading FR in daily (24-hour) blocks towards trading in 4-hour blocks (the so-called Electricity Forward Agreement (EFA) time blocks), for each of which the market participants submit a single FR offer, applying along the whole duration of the block [6]. This move has been driven by scientific evidence that a finer trading resolution can better capture the differentiated FR requirements along the day (according to the temporal profiles of demand and RES output) and thus reduce the total operating costs of the power system [7], [8]. The most prominent development is the introduction of multiple FR products [9], traded in parallel, which usually are mutually-exclusive, i. e., it is not permitted to stack them along the same time block. This exclusivity is driven by the need for measuring the delivered FR power as the difference between a baseline and the actual metered power output [10]-[12]. The fundamental distinction across these products lies in their activation source. Some of them refer to very fast response (< 1 s) and low utilization factors, while others allow slower response (~10 s) but are usually associated with high utilization factors. It is noted that we use the term utilization factor to denote the ratio ([0, 1]) of the capacity committed for FR, which is actually deployed by the system operator in real time and thus amounts to delivered (absorbed from or injected to the grid) energy. Therefore, the optimal selection among them is driven by a trade-off between their market prices and their energy intensity, expressed via their utilization factors. Although stacking among these products is prohibited, the participants may combine their offering with that in the energy markets.

Given the very fast response capabilities of various energy storage technologies, especially of batteries, such technologies may provide a multitude of market products, some of which are merely interacting (i. e., energy products against FR products) and some of which are mutually-exclusive (i. e., FR products against each other over the same time block). The investigation of the optimal participation of storage in such a market environment corresponds to a research area that is commonly referred to, in the scientific literature, as the multi-market participation or revenue stacking of storage.

A crucial challenge of this research area lies in effectively capturing various relevant sources of uncertainty. Depending on whether the participant acts as a price-maker or a price-taker, the first source of uncertainty is related to either the rival offers, which drive the formation of prices, or the market prices themselves [13]. The second source of uncertainty is related to the utilization factors of the FR products, i. e., whether and to which extent the capacity committed for FR at the day-ahead stage will be deployed by the system operator in real time. The latter depends on the system frequency, which is a highly uncertain parameter.

In [14], the offering of multiple services is optimized by stand-alone price-taking storage, namely distribution network congestion management, energy arbitrage, reserve, and FR. The storage participant anticipates the energy and FR prices in a deterministic manner. The deliverability of FR in real time is guaranteed for each time step independently, while the time-coupling energy impact of FR utilization is not addressed. A similar approach towards the utilization factors is adopted by [15]. In [16], the optimal offering of energy and FR for a price-taking battery storage is examined, where market prices are considered through probabilistic scenarios, while the utilization factor of FR is only considered in a deterministic manner, through its expected value. A similar approach towards the utilization factors is adopted by [17] and [18]. In [19], the profit-maximization problem of stand-alone price-making storage in joint energy-reserve markets is examined. The uncertain rival (supply and demand) offers are anticipated in a deterministic manner. The deliverability of reserve is guaranteed for the worst-case scenario, i.e., for a utilization factor equal to 1 at all times.

Moving to approaches which address the uncertainty over involved parameters in a probabilistic manner [20], [21], stochastic programming is employed in [22] for optimizing the offering of a price-making virtual power plant (VPP), including energy storage, in energy and reserve markets. Both uncertain rival offers and the uncertain utilization factor of reserve are considered through probabilistic scenarios. Therefore, the deliverability of reserve is guaranteed only in case any of the anticipated scenarios is realized. A similar stochastic programming approach is used in [23], where the price-making participation of stand-alone storage in energy and reserve markets is investigated. In [24], the scheduling of price-making storage in energy and reserve markets is studied, providing probabilistic guarantees for the deliverability of reserves in real time, based on chance-constrained optimization. Relevant work in this field includes [25]-[29].

A different approach towards addressing the involved uncertainties is to employ a risk-averse, yet not necessarily over-conservative strategy via the utilization of robust optimization [30]-[32]. In [33], the optimal offering of price-taking storage in joint energy and ancillary service markets is modeled as an instance of robust optimization without recourse. The profit is maximized for the worst-case realization, within participant-defined uncertainty budgets associated with the uncertain prices and utilization factors. Relevant work in this field includes [27], [34], [35].

Finally, hybrid approaches have been used by some studies towards addressing the involved uncertainties, i.e., adopting a stochastic and risk-neutral approach with respect to some parameters, and a robust and risk-averse approach with respect to others. In particular, [36] and [37] focus on the optimal offering problem of price-taking VPPs, and provide valuable insights in the effects of the two sources of uncertainty (prices and utilization factors of FR) and subsequently in the approach best suited to each of them. Specifically, they conclude that a stochastic approach is best suited to address uncertain prices, since the uncertain prices only impact optimality of the offering problem, while a robust approach is best suited to address uncertain utilization factors, since the uncertain utilization factors impact the feasibility of the problem as well. Towards validating the argument associated with uncertain utilization factors, [37] compares a stochastic approach against a robust approach, demonstrating that the latter outperforms the former in guaranteeing deliverability of reserve commitments. Similar conclusions are drawn in [27], which focuses on the optimal offering problem of a fleet of electric vehicles in day-ahead energy and FR markets, and specifically for the market setting of France. While considering perfect information on prices, three approaches for addressing uncertain utilization factors are compared, namely a deterministic approach based on their expected value, a stochastic approach, and a robust approach. It concludes that the last two approaches significantly outperform the first one in guaranteeing deliverability of FR commitments.

Although the above review indicates a rich literature in the area of multi-market participation of storage, we identify two research gaps. First of all, there is no existing paper that addresses this problem within the recent market design developments for the provision of FR products in the UK, namely trading FR products in time blocks, and trading of multiple, mutually-exclusive FR products. Secondly, although [27] and [37] compare different approaches for addressing uncertain utilization factors, they do not focus on stand-alone storage, and their comparison frameworks do not involve all approaches in the literature, namely deterministic one based on expected values, deterministic one based on worst-case values, stochastic one, and robust one. In the context of manifesting these research gaps and where our work stands within the existing literature, Table I presents the main characteristics of existing literature on participation of energy storage in energy and balancing markets.

This paper aims at addressing these two research gaps, by achieving the following contributions.

TABLE I
SUMMARY OF EXISTING LITERATURE ON PARTICIPATION OF ENERGY STORAGE IN ENERGY AND BALANCING MARKETS

Reference	Trading balancing products in blocks	Balancing products	Mutual exclusivity among balancing products	Anticipation of utilization factors
[14]	×	Multiple	×	Worst-case (time-decoupled)
[15]	×	Single	-	Worst-case (time-decoupled)
[16]	×	Multiple	×	Expected
[17], [18]	×	Single	-	Expected
[19]	×	Multiple	×	Worst-case
[22]-[24], [26], [28], [29]	×	Single	-	Stochastic
[25]	×	Multiple	×	Stochastic
[27]	×	Multiple	×	Expected, stochastic, robust
[33]	×	Multiple	×	Robust
[34]-[36]	×	Single	-	Robust
[37]	×	Single	-	Stochastic, robust
This paper	√	Multiple	√	Expected, worst-case, stochastic, robust

Note: √, ×, and - represent that the aspect is not included in, included in, and not relevant to the reference, respectively.

1) A novel optimal offering model for stand-alone price-taking energy storage participants in day-ahead energy and FR markets is proposed, accounting for the time blocks and mutual exclusivity of FR products in the UK. This model consists of two stages. The first (day-ahead) stage devises optimal offers under uncertainty and the second (real-time) stage represents the operation of storage after uncertainty is materialized. The model is presented in a generic manner with respect to the real-time stage representation, and is then specified according to 4 alternative formulations for the real-time stage, overall yielding 4 different model instances. These instances, denoted as EV, WC, SP, and RO, correspond to the different approaches in the existing literature for addressing uncertain FR utilization factors, i.e., ① deterministic approach based on their expected values, ② deterministic approach based on their worst-case values, ③ stochastic approach, through a set of scenarios, and ④ robust approach.

2) We develop a concrete methodological framework for comparing the above instances, where each instance is employed given a common training set of historical data, and the optimal offers devised by each instance are then applied to a common test set, comprising of out-of-sample realizations of the uncertain FR utilization factors. This framework enables a consistent and pragmatic comparison of the 4 instances, which is performed against 3 performance indicators: ① achieved profit, ② violation rate with respect to FR delivery, and ③ computational scalability.

3) Case studies employing real data from UK energy and FR markets demonstrate that the EV and WC instances essentially constitute the naive over-optimistic benchmark and the over-pessimistic benchmark, respectively, yielding the highest and lowest profits, and the highest and lowest FR delivery violations. On the other hand, the SP and RO instances are shown to exhibit a better trade-off between profitability and FR delivery violations, with the RO instance leaning more towards lower profits and lower violations. Furthermore, the RO instance exhibits two relative advantages with respect to the SP instance: ① better computational scalability, and ② flexibility for storage participants to adjust the trade-off between profitability and FR delivery violations, by factoring their risk appetite into their offering strategy.

The rest of this paper is organized as follows. Section II details the assumptions made in this paper. Section III presents the mathematical formulation of the proposed model. Section IV includes the implemented case studies. Finally, Section V draws the conclusions.

II. Assumptions

This paper focuses on the optimal offering problem of a stand-alone storage market participant, making the following key assumptions.

Assumption 1: the proposed model is generally applicable to any storage technology, assuming though that the examined technology has the technical capability to provide the three FR products detailed in Assumption 4 below.

Assumption 2: the storage participant behaves as a price-taker [13], [20], considering the prices of both energy and FR markets as exogenous parameters that are not impacted by its offers. Given this, the proposed model focuses on the derivation of optimal offer quantities, although the offers submitted to actual markets consist of both offer quantities and prices. For simplicity, and in order to avoid divergence from the key focus of the paper, we assume that all offers involve an offer price that guarantees acceptance; specifically: ① energy buying offers are offered at the price cap of the market, ② energy selling offers are offered at a zero price, and ③ capacity offers for FR are offered at zero price.

Assumption 3: the examined energy market is a generic pool and day-ahead market with an hourly resolution.

Assumption 4: we focus on the following 3 FR products currently traded in the UK, which are automatically activated based on system frequency deviations, rather than being activated by the system operator based on the outcomes of the imbalance market clearing.

1) Dynamic containment (DC): deployed after significant frequency deviations (± 0.2 Hz) to meet the most urgent needs for fast (<1 s) FR. It is generally characterized by the lowest utilization factors.

2) Dynamic modulation (DM): deployed during sudden and large power imbalances, where frequency moves towards the edge of the operational range (± 0.1 Hz) in a fast fashion (<1 s). It is characterized by intermediate utilization factors.

3) Dynamic regulation (DR): deployed pre-fault to correct continuous but small frequency deviations (± 0.015 Hz) in a relatively slow fashion (<10 s). It is characterized by the highest utilization factors.

Assumption 5: considering current UK market rules, these 3 FR products are traded with a day-ahead horizon and in EFA time blocks (each corresponding to 4 hours of the delivery horizon). Furthermore, each participant is allowed to submit an offer only for one of these products per each time block, i. e., mutual exclusivity is imposed or equivalently stacking is not permitted. Finally, the compensation for all 3 FR products is only capacity-based and there is no payment for actual utilization.

III. Mathematical Formulation of Proposed Model

A. Objective Function and Set of First-stage Constraints

As introduced in Section I, all 4 instances of the proposed model share a common objective function as well as a common representation of the first stage of the problem. The first-stage decisions, $V=\{p_t^{DA,S}, p_t^{DA,B}, p_e^{x,U}, p_e^{x,D}, u_e^x\}$, include all offering decisions taken in a second-stage-independent fashion, i.e., these decisions are fixed for all plausible realizations of the uncertainty at the second stage of the problem, where $p_t^{DA,S}, p_t^{DA,B}, p_e^{x,U}, p_e^{x,D} \in \mathbb{R}_{\geq 0}$, and $u_e^x \in \{0,1\}$. The proposed model constitutes a mixed-integer linear program (MILP).

$$\max_{V, V^{2ST}} \sum_t \left[\left(p_t^{DA,S} - p_t^{DA,B}\right) \sum_s \pi_s \lambda_{t,s}^{DA} \right] \Delta t + \sum_e \sum_x \left(p_e^{x,U} \sum_s \pi_s \lambda_{e,s}^{x,U} + p_e^{x,D} \sum_s \pi_s \lambda_{e,s}^{x,D} \right) \Delta e \tag{1}$$

s.t.

$$p_t^{DA,S} + \sum_x p_e^{x,U} \le \overline{P} \quad \forall e \in T_e, \forall t \in T_e \tag{2}$$

$$p_t^{DA,B} + \sum_x p_e^{x,D} \le \overline{P} \quad \forall e \in T_e, \forall t \in T_e \tag{3}$$

$$0 \le p_e^{x,U} \le u_e^x M \quad \forall e, \forall x \tag{4}$$

$$0 \le p_e^{x,D} \le u_e^x M \quad \forall e, \forall x \tag{5}$$

$$\sum_x u_e^x \le 1 \quad \forall e \tag{6}$$

$$\{V, V^{2\mathrm{ST}}\} \in \Upsilon^{2\mathrm{ST}} \tag{7}$$

where $V^{2\mathrm{ST}}$ is a generic representation of the second-stage decisions, to be specified in Section III-B; and $\Upsilon^{2\mathrm{ST}}$ is the set of the second-stage constraints. The objective function (1) maximizes the expected profit from energy and FR offers over the whole day. Constraints (2)-(6) only involve the first-stage decisions and represent the rule-set of the examined markets for the submitted offers. Constraints (2) and (3) impose the maximum limits to the combination of energy offers and offers for upward and downward provisions of FR products, respectively. In case an energy selling offer is made to the energy market during time period t, (2) limits the available capacity to be offered for upward provision to the difference between the maximum discharging power and the energy selling offer. In case an energy buying offer (or no offer) is made during time period t, (2) limits the available capacity to be offered for upward provision to the maximum discharging power. That is, a real-time change of direction from absorbing to injecting power is allowed to happen as long as $p_e^{x,U} \le \overline{P}$ holds. For example, in a case where the energy baseline is to buy energy from the market ($p_t^{DA,B} > 0$ and $p_t^{DA,S} = 0$), the real-time direction may refer to injecting power to the grid if $p_e^{x,U} > p_t^{DA,B}$. Constraint (3) acts in an equivalent manner with (2) for the case of downward provision. The combination of constraints (4) - (6) ensures that stacking of multiple FR products in the same time block is prohibited, by employing binary variables u_e^x, which express whether the storage participant makes an offer for a particular FR product in a particular time block. Constraint (7) expresses, in a generic form, that all decisions must be feasible with respect to the generic set of the second-stage constraints, which is to be specified according to the alternative model instances detailed in Section III-B.

B. Set of the Second-stage Constraints

As introduced in Section III-A, 4 alternative formulations of the second stage are examined according to the approach adopted to address uncertain FR utilization factors, yielding 4 different instances of the proposed model.

1) Anticipation of FR Utilization Factors Through Their Expected Values (EV Instance)

In this instance, the storage participant anticipates the uncertain FR utilization factors in a deterministic manner through their expected values. In practice, these expected values should be determined by the storage participant based on the average values of the utilization factors across a number of preceding days. The set of the second-stage constraints $\Upsilon^{2\mathrm{ST}}$ can be expressed as:

$$\begin{cases} p_t^{ch} \le \overline{P} \\ p_t^{dch} \le \overline{P} \end{cases} \quad \forall t \tag{8}$$

$$(p_t^{DA,S} - p_t^{DA,B})\Delta t + \sum_x \sum_w (\pi_w \alpha_{t,w}^{x,U}) p_e^{x,U} - \sum_x \sum_w (\pi_w \alpha_{t,w}^{x,D}) p_e^{x,D} = (p_t^{dch} - p_t^{ch})\Delta t \quad \forall e \in T_e, \forall t \in T_e \tag{9}$$

$$SOC_t = SOC_{t-1} + n^{ch} p_t^{ch} \Delta t - \frac{p_t^{dch}}{n^{dch}} \Delta t \quad \forall t \tag{10}$$

$$\underline{SOC} \le SOC_t \le \overline{SOC} \quad \forall t \tag{11}$$

$$SOC_{|T|} \ge SOC_0 \tag{12}$$

where $V^{2\mathrm{ST}} = V^{EV} = \{p_t^{ch}, p_t^{dch}, SOC_t\}$ includes all operational decisions of storage taken after the realization of uncertainty, and $p_t^{ch}, p_t^{dch}, SOC_t \in \mathbb{R}_{\ge 0}$. However, since the uncertain utilization factors are considered through their expected values, the second-stage operational decisions essentially remain deterministic. Constraint (8) imposes upper limits to the charging and discharging power. Constraint (9) imposes the energy balance among the energy sold/bought in the energy market, the energy absorbed/injected as a result of FR utilization, and the discharging/charging energy. Parameters $\sum_w \pi_w \alpha_{t,w}^{x,U}$ and $\sum_w \pi_w \alpha_{t,w}^{x,D}$ express the expected value of the utilization factors of the upward and downward provision components of FR product x during time period t, respectively. Constraint (10) expresses the relation between charging/discharging power and state of charge. Constraint (11) enforces that the state of charge remains within the feasible lower and upper limits. Constraint (12) ensures that the final state of charge is higher or equal to the initial one.

2) Anticipation of FR Utilization Factors Through Their Worst-case Values (WC Instance)

In this instance, the storage participant still anticipates the uncertain FR utilization factors in a deterministic manner, but now through their worst-case values (equal to 1). This implies that, in contrast to the EV instance, historical FR utilization data are irrelevant. The set of second-stage constraints $\Upsilon^{2\mathrm{ST}}$ can be expressed as:

$$SOC_t^{\max} = SOC_0 + \sum_{o=1}^{e} \sum_{i \in T_o | i \le t} \left(-\frac{p_i^{DA,S}}{n^{dch}} \Delta t + n^{ch} p_i^{DA,B} \Delta t \right) + n^{ch} \Delta t \sum_{o=1}^{e} \sum_{i \in T_o | i \le t} \sum_x p_o^{x,D} \le \overline{SOC} \quad \forall e \in T_e, \forall t \in T_e \tag{13}$$

$$SOC_t^{\min} = SOC_0 + \sum_{o=1}^{e} \sum_{i \in T_o | i \le t} \left(-\frac{p_i^{DA,S}}{n^{dch}} \Delta t + n^{ch} p_i^{DA,B} \Delta t \right) - \frac{\Delta t}{n^{dch}} \sum_{o=1}^{e} \sum_{i \in T_o | i \le t} \sum_x p_o^{x,U} \ge \underline{SOC} \quad \forall e \in T_e, \forall t \in T_e \tag{14}$$

$$SOC_{|T|}^{\min} \ge SOC_0 \tag{15}$$

where $V^{2ST} = V^{WC} = \{SOC_t^{\max}, SOC_t^{\min}\}$ includes all operational decisions of storage taken after the realization of uncertainty, and $SOC_t^{\max}, SOC_t^{\min} \in \mathbb{R}_{\ge 0}$. Due to the worst-case anticipation of uncertainty, these (deterministic) decisions may now be expressed merely as a function of the first-stage decisions. Constraint (13) expresses that during each time period t, the maximum possible value of the state of charge $SOC_t^{\max}$

is calculated by aggregating the energy buying/selling commitments, as well as the downward provision commitments (implying their full utilization), of all time periods i preceding t ($i \le t$). Moreover, it enforces that the value of $SOC_t^{\max}$ must be lower or equal to the maximum feasible state of charge at all time. Constraint (14) expresses that during each time period t, the minimum possible value of the state of charge $SOC_t^{\min}$ is calculated by aggregating the energy buying/selling commitments, as well as the upward provision commitments (implying their full utilization), of all time periods i preceding t ($i \le t$). Moreover, it enforces that the value of $SOC_t^{\min}$ must be higher or equal to the minimum feasible state of charge at all time. Constraint (15) ensures that the final state of charge is higher or equal to the initial one.

3) Anticipation of FR Utilization Factors Through a Set of Scenarios (SP Instance)

In this instance, the storage participant anticipates the uncertain FR utilization factors in a stochastic manner through a set of scenarios. In practice, each of these scenarios should be determined based on a number of preceding days. The set of second-stage constraints $\Upsilon^{2\mathrm{ST}}$ can be expressed as:

$$p_{t,w}^{ch} \le z_{t,w}^{ch} \overline{P} \quad \forall t, \forall w \tag{16}$$

$$p_{t,w}^{dch} \le (1 - z_{t,w}^{ch}) \overline{P} \quad \forall t, \forall w \tag{17}$$

$$(p_t^{DA,S} - p_t^{DA,B})\Delta t + \sum_x \alpha_{t,w}^{x,U} p_e^{x,U} - \sum_x \alpha_{t,w}^{x,D} p_e^{x,D} = (p_{t,w}^{dch} - p_{t,w}^{ch})\Delta t \quad \forall e \in T_e, \forall t \in T_e, \forall w \tag{18}$$

$$SOC_{t,w} = SOC_{t-1,w} + n^{ch} p_{t,w}^{ch} \Delta t - \frac{p_{t,w}^{dch}}{n^{dch}} \Delta t \quad \forall t, \forall w \tag{19}$$

$$\underline{SOC} \le SOC_{t,w} \le \overline{SOC} \quad \forall t, \forall w \tag{20}$$

$$SOC_{|T|,w} \ge SOC_0 \quad \forall w \tag{21}$$

where $V^{2\mathrm{ST}} = V^{SP} = \{p_{t,w}^{ch}, p_{t,w}^{dch}, SOC_{t,w}, z_{t,w}^{ch}\}$ includes all operational decisions of energy storage taken after the realization of uncertainty, i.e., these decisions are now taken per scenario and are thus indexed by w, and $p_{t,w}^{ch}, p_{t,w}^{dch}, SOC_{t,w} \in \mathbb{R}_{\ge 0}$, $z_{t,w}^{ch} \in \{0,1\}$. Constraints (16) and (17) impose upper limits to the charging and discharging power, respectively, and also ensure that simultaneous charging and discharging is prevented by employing a relevant binary variable $z_{t,w}^{ch}$; the latter is required since the consideration of uncertainty may yield such simultaneous charging and discharging if the original convex operating model of energy storage is employed [38]. Constraints (18)-(21) are similar to (9)-(12), respectively, but are defined per scenario.

Given the above formulation, the devised offers are feasible for any realization of the uncertain FR utilization factors that is included in the considered scenario set W; however, this does not generally hold for any other scenario not included in W. Such potential infeasibility effects naturally raise concerns by the storage participants, associated with fiscal implications (i.e., FR non-delivery penalties) or even prohibition of their future participation in FR markets by the system operator. Moreover, the extent to which real-time feasibility is achieved depends on the number of considered scenarios within W, implying a trade-off between real-time feasibility and computational requirements.

4) Anticipation of FR Utilization Factors Through a Risk-averse Approach (RO Instance)

To address the concerns arising from the SP instance, we examine an instance which guarantees the real-time deliverability of FR commitments for any plausible realization of the uncertain FR utilization factors, yet within certain uncertainty sets that reflect confidence bounds of the storage participant. In practice, these uncertainty sets should be determined by the storage participant based on observations of extreme utilization contingencies in preceding days, combined with its risk appetite. The set of the second-stage constraints $\Upsilon^{2\mathrm{ST}}$ can be expressed as (22), (23), and (15).

$$-SOC_t^{\max} = -SOC_0 - \sum_{o=1}^{e} \sum_{i \in T_o | i \le t} \left(-\frac{p_i^{DA,S}}{n^{dch}} \Delta t + n^{ch} p_i^{DA,B} \Delta t \right) + n^{ch} \cdot \min_{\alpha_{i,t}^{x,D} \in C^{LL}} \left\{ \sum_{o=1}^{e} \sum_{i \in T_o | i \le t} \sum_x (-\alpha_{i,t}^{x,D} p_o^{x,D}) \right\} \ge -\overline{SOC} \quad \forall e \in T_e, \forall t \in T_e \tag{22}$$

$$SOC_t^{\min} = SOC_0 + \sum_{o=1}^{e} \sum_{i \in T_o | i \le t} \left(-\frac{p_i^{DA,S}}{n^{dch}} \Delta t + n^{ch} p_i^{DA,B} \Delta t \right) + \frac{1}{n^{dch}} \cdot \min_{\alpha_{i,t}^{x,U} \in C^{LL}} \left\{ \sum_{o=1}^{e} \sum_{i \in T_o | i \le t} \sum_x (-\alpha_{i,t}^{x,U} p_o^{x,U}) \right\} \ge \underline{SOC} \quad \forall e \in T_e, \forall t \in T_e \tag{23}$$

where $V^{2\mathrm{ST}} = V^{RO} = \{SOC_t^{\max}, SOC_t^{\min}\}$. Constraints (22) and (23) are similar to (13) and (14), respectively, but enforce that the values of $SOC_t^{\max}$ and $SOC_t^{\min}$ respect the maximum and minimum feasible states of charge for any plausible realization of the uncertain variables $\alpha_{i,t}^{x,D}$ and $\alpha_{i,t}^{x,U}$, respectively, constrained by the uncertainty set C^{LL} (instead of an unconstrained worst case considered in the WC instance of Section III-B-2)). For constraint (22), during each time period t, we devise $\alpha_{i,t}^{x,D}$ through an inner problem, which defines the maximum value of $SOC_t^{\max}$, constrained by C^{LL}. For constraint (23), during each time period, we devise $\alpha_{t,w}^{x,U}$ through an inner problem, which defines the minimum value of $SOC_t^{\min}$, constrained by C^{LL}. In essence, the inner variables $\alpha_{i,t}^{x,D}$ and $\alpha_{t,w}^{x,U}$ introduced in this instance express the worst-case (constrained by C^{LL}) values of the upward and downward utilization factors during time period i, respectively, retrieved by the solution of the inner problems associated with target period t.

The uncertainty set C^{LL} is defined as:

$$C^{LL} = \{\alpha_{i,t}^{x,U}, \alpha_{i,t}^{x,D} : (25)\text{-}(28)\} \tag{24}$$

$$\alpha_{i,t}^{x,U} \le 1 : \rho_{i,t}^{x,U} \quad \forall t, \forall i \le t, \forall x \tag{25}$$

$$\alpha_{i,t}^{x,D} \le 1 : \rho_{i,t}^{x,D} \quad \forall t, \forall i \le t, \forall x \tag{26}$$

$$\sum_{i \in T_o | i \le t} \alpha_{i,t}^{x,U} \le UT_o^{x,U} : \xi_o^{x,U} \quad \forall e \in T_e, \forall t \in T_e, \forall o \le e, \forall x \tag{27}$$

$$\sum_{i \in T_o | i \le t} \alpha_{i,t}^{x,D} \le UT_o^{x,D} : \xi_o^{x,D} \quad \forall e \in T_e, \forall t \in T_e, \forall o \le e, \forall x \tag{28}$$

where $\xi_{t,s}^{x,U}, \xi_{t,s}^{x,D}, \rho_{i,t,s}^{x,U}, \rho_{i,t,s}^{x,D} \in \mathbb{R}_{\le 0}$ constitute the dual variables of the above constraints. Constraints (25) and (26) express physical limits, as the maximum utilization factor of any FR product naturally equals 1. The key element of the proposed model is the introduction of participant-defined uncertainty budgets, $UT_e^{x,U}$ and $UT_e^{x,D}$, which express the maximum sum of utilization factors across an entire time block for upward and downward provisions, respectively, for which the devised decisions must be feasible. Hence, this instance guarantees the real-time deliverability for any combination of offered FR products and across all time blocks, as long as the time-block-wide sum of utilization factors of each of these products is bounded by $UT_e^{x,U}$ and $UT_e^{x,D}$. This instance becomes equivalent to the WC instance by setting $UT_e^{x,U}= UT_e^{x,D}=|T_e|=4$ for all FR products and all time blocks. In the RO instance, the storage participant defines the values within the range (0,4) where a lower/higher value indicates a more optimistic/pessimistic participant with respect to the anticipated FR utilization. Furthermore, the participant may define different values for different products and for upward and downward provisions of a particular product according to the available data/expectations.

Given the above formulation, the devised offers are feasible for any realization of the uncertain FR utilization factors within the uncertainty set C^{LL}. In contrast with the SP instance, this instance: ① provides guarantees of real-time deliverability without assuming knowledge of the probability density functions of the utilization factors; ② constitutes an intuitive and pragmatic method for storage participants, as they merely need to express their risk appetite by setting accordingly the values of $UT_e^{x,U}$ and $UT_e^{x,D}$; and ③ does not entail a trade-off between conservativeness and computational requirements.

C. Out-of-sample Validation

This subsection details the approach we adopt for validating the offering decisions devised by each of the 4 model instances against out-of-sample scenarios of the uncertain FR utilization factors. The term out-of-sample implies that the values of the FR utilization factors in these scenarios have not been necessarily included in the dataset input to the optimal offering instances of Section III-B. For each out-of-sample scenario $r \in R$, denoting the given realized market prices and FR utilization factors with a subscript r, the operation of the storage participant is optimized according to the problem (29)-(34). It is noted that the offers devised by the model instances of Section III-B constitute fixed parameters for this problem, and are thus denoted in bold font.

$$\min_{p_{t,r}^{ch}, p_{t,r}^{dch}, SOC_{t,r}, z_{t,r}^{ch}, l_{t,r}^{+}, l_{t,r}^{-}} \sum_{t \in T} (l_{t,r}^{+} + l_{t,r}^{-})\Delta t \tag{29}$$

s.t.

$$p_{t,r}^{ch} \le z_{t,r}^{ch} \overline{P} \quad \forall t \tag{30}$$

$$p_{t,r}^{dch} \le (1 - z_{t,r}^{ch}) \overline{P} \quad \forall t \tag{31}$$

$$(\boldsymbol{p}_t^{DA,S} - \boldsymbol{p}_t^{DA,B})\Delta t + \sum_x \alpha_{t,r}^{x,U} \boldsymbol{p}_e^{x,U} - \sum_x \alpha_{t,r}^{x,D} \boldsymbol{p}_e^{x,D} = (p_{t,r}^{dch} - p_{t,r}^{ch} + l_{t,r}^{+} - l_{t,r}^{-})\Delta t \quad \forall e \in T_e, \forall t \in T_e \tag{32}$$

$$SOC_{t,r} = SOC_{t-1,r} + n^{ch} p_{t,r}^{ch} \Delta t - \frac{p_{t,r}^{dch}}{n^{dch}} \Delta t \quad \forall t \tag{33}$$

$$\underline{SOC} \le SOC_{t,r} \le \overline{SOC} \quad \forall t \tag{34}$$

where $p_{t,r}^{ch}, p_{t,r}^{dch}, SOC_{t,r}, l_{t,r}^{+}, l_{t,r}^{-} \in \mathbb{R}_{\ge 0}$ and $z_{t,r}^{ch} \in \{0, 1\}$. The objective function (29) expresses the aim of the storage participant to minimize the sum of over-delivery and under-delivery violations with respect to the delivery of FR products over the examined day. Constraints (30)-(34) express the operating constraints of storage that have been previously discussed.

Given the operating decisions of storage $\{\boldsymbol{p}_{t,r}^{ch}, \boldsymbol{p}_{t,r}^{dch}, \boldsymbol{SOC}_{t,r}, \boldsymbol{l}_{t,r}^{+}, \boldsymbol{l}_{t,r}^{-}\}$ determined by the above problem, the total profit achieved in scenario r can be calculated as:

$$Profit_r = \sum_t [(\boldsymbol{p}_t^{DA,S} - \boldsymbol{p}_t^{DA,B})\lambda_{t,r}^{DA}]\Delta t + \sum_e \sum_x (\boldsymbol{p}_e^{x,U} \lambda_{e,r}^{x,U} + \boldsymbol{p}_e^{x,D} \lambda_{e,r}^{x,D})\Delta e \tag{35}$$

The total energy violation with respect to the delivery of FR products can be calculated as:

$$Violation_r = \sum_t (\boldsymbol{l}_{t,r}^{+} + \boldsymbol{l}_{t,r}^{-})\Delta t \tag{36}$$

The violation rate as the ratio between the energy violation and the energy that should have been delivered can be calculated as:

$$VR_r = \frac{Violation_r}{\sum_e \sum_{t \in T_e} \sum_x \left| \alpha_{t,r}^{x,U} \boldsymbol{p}_e^{x,U} - \alpha_{t,r}^{x,D} \boldsymbol{p}_e^{x,D} \right|} \times 100\% \tag{37}$$

The energy throughput can be calculated as:

$$Throughput_r = \sum_t \frac{\boldsymbol{p}_{t,r}^{dch}}{n^{dch}} \Delta t \tag{38}$$

And the energy throughput expressed in terms of cycles [39] can be expressed as:

$$Cycles_r = \frac{Throughput_r}{\overline{SOC} - \underline{SOC}} \tag{39}$$

IV. Case Studies

A. Description

The case studies aim at applying the proposed model for an examined storage participant and comparing its 4 instances based on real historical data from UK energy and FR markets. Specifically, we focus on the period 07/05/2022-31/01/2023, with the test set (employed for the out-of-sample validation of the 4 instances) including the period 24/10/2022-31/01/2023 (i. e., 100 days in total). For each day d in this test set, we devise the offering decisions by each instance based on an associated training set (in-sample scenarios), which includes the N days preceding day d, and our analysis includes a sensitivity analysis on N. For example, for $d=$ 01/01/2023 and $N=31$, the training set includes the period 01/12/2022-31/12/2022. In all 4 instances, the expected values of energy and FR prices are determined by their average value across the last $|S|=10$ days of the trainning set. The uncertain FR utilization factors are addressed by each instance as follows.

1) EV: the expected values of FR utilization factors are determined by their average values across the N days of the training set.

2) WC: since we anticipate the FR utilization factors through their worst-case values (equal to 1), their historical values are irrelevant.

3) SP: each of the considered scenarios corresponds to one day of the training set, i.e., $|W|=N$.

4) RO: the uncertainty budgets $UT_e^{x,U}$ and $UT_e^{x,D}$ are determined based on the maximum observed time-block-wide sum of utilization factors for upward and downward provisions, respectively, of product x during time block e, over the training set.

The assumed operating parameters of the examined storage participant are: $\overline{P}=50$ MW, $\underline{SOC}=5$ MWh, $\overline{SOC}=100$ MWh, $SOC_0=5$ MWh, $n^{ch}=n^{dch}=0.9$. The historical energy prices and FR prices for our focused period have been derived from [39] and [40], respectively. Concerning historical FR utilization factors, the system frequency data in [41] are used and converted to utilization factors, as per the relevant functions found in [10]-[12]. Table II presents the average prices, the average utilization factors, and the maximum utilization factors of each FR product and each direction of provision (upward and downward) over the test set. All studies are executed using Gurobi [42], on a computer with a 4-core 3.4 GHz Intel(R) Core(TM) i7-6700 processor and 16 GB of RAM.

TABLE II
PRICES AND UTILIZATION FACTORS OF FR PRODUCTS OVER TEST SET

FR product	Average price (£/MW/h)	Average utilization factor (MWh/MW)	Maximum utilization factor (MWh/MW)
DC upward	6.14	0.0058	0.0269
DM upward	1.73	0.0159	0.1118
DR upward	12.60	0.1150	0.5389
DC downward	3.26	0.0056	0.0274
DM downward	5.61	0.0148	0.1215
DR downward	5.66	0.1112	0.5414

B. Results

Tables III-VI present the performance of each of the 4 instances, including 5 performance indicators: ① total profit achieved by the storage participant, as determined by (35) (also broken down to its energy and FR components); ② volume of FR offers, broken down to each FR product and each direction of provision; ③ violation rate with respect to the delivery of FR products, as determined by (37); ④ storage cycles, as determined by (39); and ⑤ computational time required for solving the offering problem. These performance indicators are averaged over the number of days included in the test set. Furthermore, we carry out a sensitivity analysis on the number N of preceding days included in the training set.

TABLE III
PERFORMANCE OF EV INSTANCE

N	Average profit (£)			Average volume of FR offers (MW/day)						Average violation rate (%)	Average storage cycles per day	Average computational time (s)
	Energy	FR	Total	DC upward	DM upward	DR upward	DC downward	DM downward	DR downward			
10	24623	12505	37128	60	6	356	16	5	1063	28.13	1.16	0.1
30	24723	12494	37217	64	12	349	14	8	1057	29.63	1.18	0.1
50	24642	12516	37158	64	18	351	14	13	1050	30.09	1.19	0.1
100	24326	12829	37155	70	18	362	15	12	1047	30.44	1.18	0.1
170	24000	13037	37037	76	18	371	18	13	1040	31.40	1.19	0.1

TABLE IV
PERFORMANCE OF WC INSTANCE

N	Average profit (£)			Average volume of FR offers (MW/day)						Average violation rate (%)	Average storage cycles per day	Average computational time (s)
	Energy	FR	Total	DC upward	DM upward	DR upward	DC downward	DM downward	DR downward			
10	9552	0	9552	0	0	0	0	0	0	0	1.12	0.1
30	9552	0	9552	0	0	0	0	0	0	0	1.12	0.1
50	9552	0	9552	0	0	0	0	0	0	0	1.12	0.1
100	9552	0	9552	0	0	0	0	0	0	0	1.12	0.1
170	9552	0	9552	0	0	0	0	0	0	0	1.12	0.1

TABLE V
PERFORMANCE OF SP INSTANCE

N	Average profit (£)			Average volume of FR offers (MW/day)						Average violation rate (%)	Average storage cycles per day	Average computational time (s)
	Energy	FR	Total	DC upward	DM upward	DR upward	DC downward	DM downward	DR downward			
10	15585	12081	27666	149	44	255	79	106	890	4.75	0.95	1
30	14563	10985	25548	175	60	155	107	164	802	1.63	0.86	20
50	13497	11094	24591	211	46	154	126	139	805	1.39	0.81	62
100	12799	10549	23348	172	54	147	96	122	847	0.83	0.71	575
170	12790	9919	22709	202	123	86	125	170	759	0.77	0.66	1192

TABLE VI
PERFORMANCE OF RO INSTANCE

N	Average profit (£)			Average volume of FR offers (MW/day)						Average violation rate (%)	Average storage cycles per day	Average computational time (s)
	Energy	FR	Total	DC upward	DM upward	DR upward	DC downward	DM downward	DR downward			
10	4343	13298	17641	474	167	201	345	351	312	1.00	0.80	0.1
30	4712	12161	16873	492	159	132	355	368	296	1.00	0.79	0.1
50	4946	11530	16476	498	137	108	370	374	276	0.86	0.81	0.1
100	5108	11303	16411	493	128	100	366	385	270	0.67	0.81	0.1
170	5114	11098	16212	501	118	93	367	387	265	0.40	0.80	0.1

Starting from the EV instance, we observe that its achieved total profits are the highest among the 4 instances for any value of N. However, it also exhibits extreme (and much higher than any other instance) violation rates of FR delivery. Therefore, it is inapplicable in real applications, since such extreme violation rates will eventually yield extreme FR non-delivery penalties for the storage participant, or even prohibition of its participation in FR markets by the system operator. In other words, this instance merely provides a naive over-optimistic (with respect to profitability) benchmark, since it makes the unrealistic assumption that FR utilization factors are known at the offering stage. We also observe that the largest volume of the FR offers corresponds to the DR product and particularly at the downward direction. This is driven by two effects: ① since this naive instance assumes that the FR utilization factors are known at the offering stage, it implicitly sets a largest weight on the prices rather than the utilization factors of the FR products (and DR generally exhibits the highest prices, as indicated in Table II); and ② the utilization of downward FR (mainly DR as explained above) leads to an increase of the state of charge, which can be exploited as additional energy sales and profits in the energy market. However, the DR product is also characterized by the highest utilization factors (as indicated in Table II), which drives the extreme violation rates of this instance. The overoptimistic nature of this instance is also reflected in the higher number of storage cycles compared with the other three instances, since its offering strategy is less restrained by the uncertain utilization factors.

Moving to the WC instance, we observe that not only its achieved total profits are the lowest among the 4 instances, but also its devised offers involve participation only in the energy market and no participation in the FR market. This is driven by the purely pessimistic perspective of this instance with respect to the FR utilization factors, in combination with the fact that the energy prices are generally higher than the FR prices (at least for the focused period of the case studies). On the other hand, it exhibits (by definition) zero (and lower than any other instance) violation rates of FR delivery. In other words, it merely provides a naive over-pessimistic benchmark, since it makes the unrealistic assumption that any FR commitment will be delivered in real time. Although no FR offers are made, this instance exhibits a higher number of storage cycles compared with the SP and RO instances. This is because sole participation in the energy market implies that all contracted offers will be delivered at their entirety in real time, and no part of them gets negated by the utilization of FR at the opposite direction.

Moving to the SP instance, we firstly observe that it exhibits a better trade-off between the achieved total profits and violation rates of FR delivery, compared with the two previous naive instances. The achieved total profits are significantly higher compared with the pessimistic WC instance, while the violation rates are significantly lower compared with the optimistic EV instance. Secondly, we observe that as N increases, both the total profit and the violation rate are reduced, since real-time feasibility is imposed for a larger number of scenarios, naturally leading to the reduction of the achieved profits. Concerning downward FR, similar to the EV instance, this instance favours offering of the DR product which exhibits the highest prices (as indicated in Table II), given that the utilization of downward FR leads to an increase of the state of charge and additional profits in the energy market. Concerning upward FR though, and in contrast to the EV instance which still favours DR, the composition of FR offers under the SP instance moves towards the DC product, especially as N increases. This is because the SP instance sets a larger weight on real-time feasibility and thus the FR utilization factors (DC generally exhibits the lowest utilization factors, as indicated in Table II). Furthermore, since its offering strategy is more restrained by the uncertain utilization factors, the number of storage cycles is significantly lower compared with the EV instance. Finally, compared with the other 3 instances, this instance exhibits a worse computational scalability; as N increases, the computational time increases disproportionately. As mentioned in Section III-B-3), this implies that the application of this instance requires the storage participant to balance a trade-off between real-time feasibility and computational cost.

Moving to the RO instance, we firstly observe that it exhibits a better trade-off between total profits and violation rates (similarly to the SP instance), compared with the two naive instances. However, compared with the SP instance, this trade-off leans more towards lower profits and lower violation rates. Secondly, similar to the SP instance, we observe that as N is increased, both the total profit and the violation rate are reduced, since we effectively seek further into the past for the maximum FR utilization factors. Since the RO instance sets a larger weight on real-time feasibility and the (maximum) FR utilization factors compared with the EV and SP instances, it favours much more FR products with low utilization factors. This is clearly evident in the upward FR offers (where DC is favoured under every N) and to a certain extent in the downward FR offers (where there is a balance among the 3 FR products, while the EV and SP instances clearly favour the DR product). Finally, in contrast with the SP instance, this instance exhibits negligible computational requirements, irrespective of the value of N. Therefore, its application does not entail a trade-off between real-time feasibility and computational cost.

Moreover, as discussed in Section III-B-4), this instance provides an intuitive and pragmatic approach for storage participants to factor their risk appetite into their offering strategy by determining accordingly their uncertainty budgets. In other words, even for a fixed size of the training set N, the examined storage participant can flexibly adjust the trade-off between the total profit and violation rate by setting accordingly the value of these uncertainty budgets. In order to quantitatively demonstrate this flexibility of the RO instance, we perform a sensitivity analysis on these uncertainty budgets, considering a specific training set with $N=170$. Specifically, we explore different cases for a parameter ϕ which expresses the relative percentage of the employed uncertainty budgets with respect to their nominal values (specified in Section IV-A and employed in the cases presented in Table VI). The results of this analysis are presented in Table VII. We can observe that as ϕ is increased (i.e., as the uncertainty budgets are increased), both the total profit and the violation rate are reduced, as the storage participant adopts a more pessimistic perspective with respect to FR utilization.

TABLE VII
PERFORMANCE OF RO INSTANCE FOR $N=170$ AND DIFFERENT VALUES OF ϕ

ϕ (%)	Average profit (£)			Average violation rate (%)	Average computational time (s)
	Energy	FR	Total		
120	5957	9717	15674	0.10	0.1
110	5662	10250	15912	0.28	0.1
100	5114	11098	16212	0.40	0.1
90	4497	12199	16696	0.55	0.1
80	4418	12613	17031	1.17	0.1
70	3734	13798	17532	1.17	0.1
60	3347	14896	18243	1.39	0.1
50	3197	16003	19200	3.76	0.1

V. CONCLUSION

This paper focuses on the research area of co-optimizing the offers of stand-alone price-taking energy storage in energy and FR markets, and achieves two relevant contributions.

Firstly, it proposes a novel optimal offering model which accounts for recent FR market design developments in the UK, namely the trade of FR products in time blocks, and the mutual exclusivity among the multiple FR products. The model consists of two stages, with the first (day-ahead) one devising optimal offers under uncertainty, and the second (real-time) one representing the operation of storage after uncertainty is materialized. Secondly, this paper develops a concrete methodological framework for comparing all the different approaches existing in the literature for addressing uncertain FR utilization factors. This is achieved by providing 4 alternative formulations for the real-time stage of the proposed offering model, yielding 4 different model instances. Following that, we carry out an out-of-sample comparison of the 4 instances, which is performed against 3 performance indicators: ① achieved profit; ② violation rate with respect to FR delivery; and ③ computational scalability.

The presented case studies compare these 4 instances for an examined storage participant based on real data from UK energy and FR markets, considering the 3 FR products currently traded in the UK, namely DC, DM, and DR. The results demonstrate that the EV instance constitutes a naive over-optimistic benchmark, yielding the highest profits but also extreme violation rates of FR delivery, rendering it inapplicable in real applications where such extreme violations yield significant non-delivery penalties or even prohibition of participation in FR markets by the system operator. On the other hand, the WC instance constitutes a naive over-pessimistic benchmark, completely avoiding FR delivery violations, but yielding the lowest profits.

The SP and RO instances are shown to exhibit a better trade-off between profitability and FR delivery violations, compared with the two previous naive instances. However, the trade-off achieved by the RO instance leans more towards lower profits and lower violations. Furthermore, the RO instance exhibits two relative advantages with respect to the SP instance. Firstly, the RO instance exhibits negligible computational requirements, while the computational requirements of the SP instance increase disproportionately with the number of considered scenarios. Secondly, the RO instance provides an intuitive and pragmatic approach for storage participants to flexibly adjust the trade-off between profitability and FR delivery violations, by factoring their risk appetite into their offering strategy.

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