A Projection and Decomposition Approach for Multi-agent Coordinated Scheduling in Power Systems

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***Abstract*—Aiming at multi-agent coordinated scheduling problems in power systems under uncertainty, a generic projection and decomposition (P&D) approach is proposed in this letter. The canonical min-max-min two-stage robust optimization (TSRO) model with coupling constraints is equivalent to a concise robust optimization (RO) model in the version of mixed-integer linear programming (MILP) via feasible region projection. The decentralized decoupling of the non-convex MILP problem is realized through a dual decomposition algorithm, which ensures the fast convergence to a high-quality solution in the distributed optimization. Numerical tests verify the superior performance of the proposed P&D approach over the existing distributed TSRO method.**

***Index Terms*—Coordinated scheduling, multi-agent system, distributed robust optimization, projection and decomposition.**

I. Introduction

With the growing penetration of renewables, its strong uncertainty brings considerable challenges to power system scheduling. In addition, due to the coexistence of multiple agents in power systems, the efficient coordination of independent stakeholders considering privacy-preserving has become the focus of the current research [1]. Distributed two-stage robust optimization (TSRO) models have been investigated for multi-agent coordinated scheduling under uncertainty [2].

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At present, the traditional distributed TSRO method deals with uncertainty by constructing a centralized min-max-min RO model. The augmented Lagrangian method, e. g., the alternating direction method of multipliers (ADMM) [3] or the analysis target cascading (ATC) method [4], further decouples the centralized TSRO model in a distributed manner. The above method has been applied to a few coordinated scheduling problems [2] - [5]. Nonetheless, some limitations exist in the application of the traditional distributed TSRO method.

1) TSRO is generally solved by an iterative algorithm, such as the column and constraint generation (C&CG) algorithm or the Benders decomposition. The iterative calculations could hardly satisfy the computational requirements in practical cases due to their large calculation expense. Such a burden grows particularly heavy if the conventional iterative solution algorithm is embedded in distributed decomposition [6]. The TSRO and the iterative solution algorithm would be therefore inappropriate to execute online distributed calculation for scheduling.

2) The representative augmented Lagrangian-based distributed methods can equivalently decouple convex centralized models, and such methods could derive the identical results as the centralized optimization [7]. However, the TSRO model features the programming in a min-max-min formulation, and the computational optimality and convergence cannot be warranted for such complex models using conventional distributed methods [8]. Although some techniques have been devised to address the convergence and optimality issues [9], they are not universally efficient, especially when coupling constraints exist in both stages of a TSRO model. As the computational performance of the traditional distributed TSRO model could hardly fulfill the requirements of practical applications, there arises a need to bring a concise yet strong approach for distributed decomposition of TSRO scheduling problems in power systems.

In view of the above research gaps, this letter proposes a novel projection and decomposition (P&D) approach for multi-agent coordinated scheduling problems under uncertainty. Compared to the traditional distributed TSRO method, the proposed P&D approach identifies a high-quality solution with low computational cost and strong scalability.

II. General Model of Centralized TSRO

The centralized TSRO model for multi-agent scheduling with uncertainty in power systems is written in matrices as (1). The compact model M_1 minimizes the total operation objective for system scheduling in the predicted nominal scenario, and enforces the power balance under uncertainty. The feasibility of these basic scheduling plans in the nominal scenario is ensured for all uncertainty scenarios, thus keeping the safe operation of the system [10].

M_1:

$$\begin{cases} \min\limits_{\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_m \in \Phi(\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_m)} \sum\limits_{i=1}^{m} \boldsymbol{c}_i^{\mathrm{T}} \boldsymbol{x}_i \\ \text{s.t. } \boldsymbol{B}_i \boldsymbol{x}_i \le \boldsymbol{e}_i \quad \forall i \\ \quad \sum\limits_{i=1}^{m} \boldsymbol{C}_i \boldsymbol{x}_i \le \boldsymbol{f} \\ \quad \Phi(\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_m) = \max\limits_{\substack{\boldsymbol{u}=\{\boldsymbol{u}_1,\boldsymbol{u}_2,\ldots,\boldsymbol{u}_m\} \\ \boldsymbol{u}_i \in \Omega_i, \forall i}} \min\limits_{\substack{\boldsymbol{y}=\{\boldsymbol{y}_1,\boldsymbol{y}_2,\ldots,\boldsymbol{y}_m\} \\ \boldsymbol{\xi}=\{\boldsymbol{\xi}_1,\boldsymbol{\xi}_2,\ldots,\boldsymbol{\xi}_m\}}} \sum\limits_{i=1}^{m} \boldsymbol{1}^{\mathrm{T}} \boldsymbol{\xi}_i \\ \quad \boldsymbol{G}_i \boldsymbol{y}_i + \boldsymbol{\xi}_i \le \boldsymbol{h}_i - \boldsymbol{L}_i \boldsymbol{u}_i - \boldsymbol{M}_i \boldsymbol{x}_i \quad \forall i \\ \quad \sum\limits_{i=1}^{m} \boldsymbol{K}_i \boldsymbol{y}_i \le \boldsymbol{g} \end{cases} \tag{1}$$

where $\boldsymbol{x}_i$ denotes the first-stage variables of agent i before uncertainty, that is, the basic scheduling plans, including the binary commitments and the continuous unit outputs; $\boldsymbol{c}_i^{\mathrm{T}} \boldsymbol{x}_i$ denotes the local operation objective of agent i in the nominal scenario, and the total operation objective of all agents $i = 1, 2, \ldots, m$ is minimized in the first-stage optimization. The second line of M_1 represents the local constraints of agent i in the nominal scenario, mainly containing the unit operation and network security within its jurisdiction, and the third line denotes the coupling constraints among agents, such as the consistency for shared line power. In the fourth line, $\Phi(\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_m)$ denotes the second-stage optimization model; and $\boldsymbol{y}_i$ denotes the continuous recourse variables of agent i after uncertainty. The second-stage recourse plans modify $\boldsymbol{x}_i$ to hedge the uncertainty. $\boldsymbol{\xi}_i$ denotes the non-negative slack variables introduced to characterize power imbalance with two parts ($\boldsymbol{\xi}_i^+, \boldsymbol{\xi}_i^-$). Note that when there is an inflow power imbalance, $\boldsymbol{\xi}_i^- > \boldsymbol{0}$ and $\boldsymbol{\xi}_i^+ = \boldsymbol{0}$; otherwise, when an outflow power imbalance emerges, $\boldsymbol{\xi}_i^+ > \boldsymbol{0}$ and $\boldsymbol{\xi}_i^- = \boldsymbol{0}$. $\boldsymbol{u}_i$ denotes the uncertainty decision variables of agent i, e.g., the uncertainty in source-load power; and Ω_i denotes the uncertainty set, such as the boxed or polyhedral set. The fifth and sixth lines represents the local constraints of agent i and the coupling constraints between multiple agents under uncertainty, respectively. $\boldsymbol{B}_i$, $\boldsymbol{e}_i$, $\boldsymbol{C}_i$, $\boldsymbol{f}$, $\boldsymbol{G}_i$, $\boldsymbol{h}_i$, $\boldsymbol{L}_i$, $\boldsymbol{M}_i$, $\boldsymbol{K}_i$, and $\boldsymbol{g}$ are the constant matrices for the first- and second-stage constraints.

III. Proposed P&D Approach

Considering that a centralized TSRO model with coupling constraints in both stages cannot be decoupled with guaranteed optimality in distributed optimization, this section proposes a P&D approach to tackle this problem.

A. Feasible Region Projection of TSRO Model

For M_1, the decision-making of the second stage intrinsically checks the scheduling feasibility under uncertainty, indicating that with the first-stage solution $\boldsymbol{x}_i$, there exists $\boldsymbol{y}_i$ for any uncertainty scenario that ensures $\boldsymbol{\xi}_i = \boldsymbol{0}$. Hence, M_1 is recast as:

M_2:

$$\begin{cases} \min\limits_{\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_m \in \Phi(\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_m)} \sum\limits_{i=1}^{m} \boldsymbol{c}_i^{\mathrm{T}} \boldsymbol{x}_i \\ \text{s.t. } \boldsymbol{B}_i \boldsymbol{x}_i \le \boldsymbol{e}_i \quad \forall i \\ \quad \sum\limits_{i=1}^{m} \boldsymbol{C}_i \boldsymbol{x}_i \le \boldsymbol{f} \\ \quad \Phi(\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_m) = \left\{ \forall \boldsymbol{u}_i \in \Omega_i, \forall i, \exists (\boldsymbol{y}_1,\boldsymbol{y}_2,\ldots,\boldsymbol{y}_m): \right. \\ \qquad \left. \boldsymbol{M}_i \boldsymbol{x}_i + \boldsymbol{G}_i \boldsymbol{y}_i + \boldsymbol{L}_i \boldsymbol{u}_i \le \boldsymbol{h}_i, \sum\limits_{i=1}^{m} \boldsymbol{K}_i \boldsymbol{y}_i \le \boldsymbol{g} \right\} \end{cases} \tag{2}$$

Compared to M_1, the slack variables $\boldsymbol{\xi}_i$ are omitted in M_2. The max-min bi-level model in the second stage is described as a multi-dimensional feasible region of the recourses, which is defined by the inner-layer linear programming constraints with respect to uncertainty. According to the Fourier-Motzkin elimination [11], such a region in the max-min optimization for each agent represented by the constraints in the fifth line of M_2 is projected to the domain formed by $\boldsymbol{x}_i$ in the first stage and $\boldsymbol{u}_i$, so as to eliminate the second-stage variables $\boldsymbol{y}_i$ [12]. After eliminating $\boldsymbol{y}_i$, the linear constraints can be generated as:

$$\boldsymbol{A}_i^{\mathrm{X}} \boldsymbol{x}_i + \boldsymbol{A}_i^{\mathrm{U}} \boldsymbol{u}_i \le \boldsymbol{k}_i \tag{3}$$

where $\boldsymbol{A}_i^{\mathrm{X}}$, $\boldsymbol{A}_i^{\mathrm{U}}$, and $\boldsymbol{k}_i$ are the parameter matrices produced in the elimination; and i identifies the constraint set generated by the corresponding agent. The general procedures are given in [13] to clarify how to derive such kinds of constraints in (3) by the Fourier-Motzkin elimination. Some accelerating policies such as redundant constraint filtering could be employed to enhance the computational performance of the Fourier-Motzkin elimination [14]. Furthermore, considering that the recourses and second-stage constraints are built by series, the Fourier-Motzkin elimination can uniformly handle each constraint series, thereby further improving its performance.

It is worth mentioning that the last term in (2) essentially represents the coupling constraints among agents only on their coupling variables in $\boldsymbol{y}_i$ under uncertainty, and it does not involve any internal variables of other agents. Such types of constraints are shared among the neighboring agents, e.g., the consistency constraints on tie-line variables between different areas in power systems. Therefore, as agent i knows these coupling variables and constraints with its neighbors, the projection for the second-stage optimization could be conducted independently by agent i without the privacy of its neighboring agents, and each agent can obtain the linear constraints in (3). A simple multi-agent optimization model with coupling constraints is provided in [13] to clarify the elimination procedures.

According to the property of the Fourier-Motzkin elimina-

tion, we can get all feasible solutions of a linear programming model from the solution results fitting these linear constraints generated by the Fourier-Motzkin elimination with two presuppositions [15]: ① the optimization of the original mathematical programming has a nonempty set; and ② the constraint coefficients of the original programming are not all zero. The specific meaning of the feasible region projection for this model is that for all agents, if $(\boldsymbol{x}_i, \boldsymbol{u}_i)$ denotes a solution set filling (3), $\boldsymbol{y}_i$ certainly exists so that $(\boldsymbol{x}_i, \boldsymbol{y}_i, \boldsymbol{u}_i)$ is a set of solutions falling into the feasible region characterized in the third and fourth lines in M_2. After the projection, M_2 is equivalently expressed as:

$$\mathrm{M}_3: \begin{cases} \min\limits_{\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_m \in \Phi(\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_m)} \sum\limits_{i=1}^{m} \boldsymbol{c}_i^{\mathrm{T}} \boldsymbol{x}_i \\ \text{s.t. } \boldsymbol{B}_i \boldsymbol{x}_i \le \boldsymbol{e}_i \quad \forall i \\ \sum\limits_{i=1}^{m} \boldsymbol{C}_i \boldsymbol{x}_i \le \boldsymbol{f} \\ \Phi(\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_m) = \{\forall \boldsymbol{u}_i \in \Omega_i, \forall i, \boldsymbol{A}_i^{\mathrm{X}} \boldsymbol{x}_i + \boldsymbol{A}_i^{\mathrm{U}} \boldsymbol{u}_i \le \boldsymbol{k}_i\} \end{cases} \tag{4}$$

M_3 is still intractable because $\Phi(\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_m)$ contains infinite uncertainty scenarios. As $\boldsymbol{u}_i$ is exogenous, to ensure (3) valid for $\forall \boldsymbol{u}_i \in \Omega_i$, the only need is to meet:

$$\boldsymbol{A}_i^{\mathrm{X}} \boldsymbol{x}_i + \max_{\boldsymbol{u}_i \in \Omega_i} \boldsymbol{A}_i^{\mathrm{U}} \boldsymbol{u}_i \le \boldsymbol{k}_i \tag{5}$$

In the above formulation, the r^{th} dimensional maximum optimization for $\boldsymbol{u}_i$ is derived as:

$$\begin{cases} \max\limits_{\boldsymbol{u}_i} [\boldsymbol{A}_i^{\mathrm{U}}]_r \boldsymbol{u}_i \\ \text{s.t. } \Omega_i = \{\boldsymbol{u}_i | \boldsymbol{C}_i^{\mathrm{U}} \boldsymbol{u}_i \le \boldsymbol{d}_i^{\mathrm{U}} : [\boldsymbol{\delta}_i]_r \ge 0\} \end{cases} \tag{6}$$

where $\boldsymbol{C}_i^{\mathrm{U}}$ and $\boldsymbol{d}_i^{\mathrm{U}}$ denote the coefficient matrices related to Ω_i of agent i; and $[\boldsymbol{\delta}_i]_r$ denotes dual variables for the r^{th} dimensional constraint. In practice, the max objectives $\boldsymbol{A}_i^{\mathrm{U}} \boldsymbol{u}_i$ can be directly determined by setting $\boldsymbol{u}_i$ as the boundaries of the uncertainty interval. For example, if $\boldsymbol{A}_i^{\mathrm{U}} > \boldsymbol{0}$ and $\boldsymbol{u}_i$ takes the upper boundaries, (5) can be satisfied, then (3) must hold for $\forall \boldsymbol{u}_i \in \Omega_i$. Without loss of generality, the duality expression for each dimension in (5) identifies the same max values.

After substituting the dual formulation of the max optimization for each dimension in (6), M_3 is transformed as:

$$\mathrm{M}_4: \begin{cases} \min\limits_{\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_m,\boldsymbol{\delta}_1,\boldsymbol{\delta}_2,\ldots,\boldsymbol{\delta}_m} \sum\limits_{i=1}^{m} \boldsymbol{c}_i^{\mathrm{T}} \boldsymbol{x}_i \\ \text{s.t. } \boldsymbol{B}_i \boldsymbol{x}_i \le \boldsymbol{e}_i \quad \forall i \\ \sum\limits_{i=1}^{m} \boldsymbol{C}_i \boldsymbol{x}_i \le \boldsymbol{f} \\ \boldsymbol{A}_i^{\mathrm{X}} \boldsymbol{x}_i + \boldsymbol{\delta}_i^{\mathrm{T}} \boldsymbol{d}_i^{\mathrm{U}} \le \boldsymbol{k}_i \quad \forall i \\ \boldsymbol{C}_i^{\mathrm{U}} \boldsymbol{\delta}_i^{\mathrm{T}} = \boldsymbol{A}_i^{\mathrm{U}} \quad \forall i \end{cases} \tag{7}$$

where $\boldsymbol{\delta}_i$ collects $[\boldsymbol{\delta}_i]_r$ by columns for all dimensions. As the Fourier-Motzkin elimination is an exact equivalent procedure, the original min-max-min TSRO model, i. e., M_1, is equivalent to a mixed-integer linear programming (MILP) problem M_4 after feasible region projection. The solution of M_4 does not need any iteration compared with M_1, which is convenient for the engineering implementation.

B. Dual Decomposition of Equivalent MILP Problem

The MILP problem M_4 after projection is refined into a compact formulation P_1.

$$\mathrm{P}_1: \begin{cases} \min\limits_{\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_m} \sum\limits_{i=1}^{m} \boldsymbol{c}_i^{\mathrm{T}} \boldsymbol{x}_i \\ \text{s.t. } \sum\limits_{i=1}^{m} \boldsymbol{A}_i \boldsymbol{x}_i \le \boldsymbol{b} \\ \boldsymbol{x}_i \in X_i, X_i = \{\boldsymbol{x}_i : \boldsymbol{D}_i \boldsymbol{x}_i \le \boldsymbol{d}_i\} \quad \forall i \end{cases} \tag{8}$$

where $\boldsymbol{\delta}_i$ in M_4 is integrated as the decision variables of agent i to form the new variable vector $\boldsymbol{x}_i$ for each agent in P_1; $\boldsymbol{A}_i$, $\boldsymbol{b}$, $\boldsymbol{D}_i$, and $\boldsymbol{d}_i$ are the constant matrices or vectors for the coupling and local constraints in the refined formulation; X_i is the compact set of all local constraints for agent i in M_4, i.e., the first, third, and fourth constraints in (7). The second line of (8) represents the corresponding coupling constraints, that is, the second constraint in (7), where the dimension of $\boldsymbol{b}$ is q.

The traditional ADMM and ATC method are less effective for an MILP because such a non-convex problem no longer meets the optimality and convergence conditions of the traditional decomposition approaches [8]. Hence, a dual decomposition algorithm is deployed for the decoupling of the projected MILP problem to guarantee the solution convergence and quality [16]. The main steps include dualizing the coupling constraints using the Lagrange multipliers and handling the Lagrangian dual programming to obtain the dual variables. Then, the primal solution is recovered by solving the local MILP of each agent given the dual variables. This guarantees the recovered primal solution satisfies the local constraints but does not ensure that the obtained primal solution satisfies the coupling constraints because of the non-convexity of MILP. Hence, the dual subgradient algorithm iteratively generates the tentative solution and updates the dual variables by each agent in a decentralized manner. By suitably averaging these tentative solutions during iterations, we could identify one solution that fits both the coupling and local constraints via the iteration between the primal and dual problems. Besides, to enforce the feasibility of the obtained dual solution, the constraint tightening strategy is adopted to restore the feasible primary solution, which shrinks the duality gap and guarantees the solution quality. A tightened version for P_1 is defined as P_2 after introducing the non-negative tightening coefficients $\boldsymbol{\rho}$.

$$\mathrm{P}_2: \begin{cases} \min\limits_{\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_m} \sum\limits_{i=1}^{m} \boldsymbol{c}_i^{\mathrm{T}} \boldsymbol{x}_i \\ \text{s.t. } \sum\limits_{i=1}^{m} \boldsymbol{A}_i \boldsymbol{x}_i \le \boldsymbol{b} - \boldsymbol{\rho} \quad \boldsymbol{\rho} \ge \boldsymbol{0}, \boldsymbol{x}_i \in X_i, X_i = \{\boldsymbol{x}_i : \boldsymbol{D}_i \boldsymbol{x}_i \le \boldsymbol{d}_i\}, \forall i \end{cases} \tag{9}$$

Its Lagrangian dual expression is formulated as:

$$\max_{\boldsymbol{\lambda} \ge \boldsymbol{0}} (-\boldsymbol{\lambda}^{\mathrm{T}} (\boldsymbol{b} - \boldsymbol{\rho})) + \sum_{i=1}^{m} \min_{\boldsymbol{x}_i \in X_i} (\boldsymbol{c}_i^{\mathrm{T}} + \boldsymbol{\lambda}^{\mathrm{T}} \boldsymbol{A}_i) \boldsymbol{x}_i \tag{10}$$

where $\boldsymbol{\lambda}$ denotes the q-dimensional dual variables; and the

definition of $\boldsymbol{\rho}$ is as follows: finding the maximal difference in the contributions of feasible solution to the coupling constraints.

$$[\rho]_j = q \cdot \max_{i \in \{1,2,\ldots,m\}} \left(\max_{\boldsymbol{x}_i \in X_i} [\boldsymbol{A}_i]_j \boldsymbol{x}_i - \min_{\boldsymbol{x}_i \in X_i} [\boldsymbol{A}_i]_j \boldsymbol{x}_i \right) \tag{11}$$

where j is the index of the matrix line in the coupling constraints.

For traditional distributed scheduling, a coordination center is set up to collect the optimal results of all shared variables among agents. Their contributions to the coupling constraints help update $\boldsymbol{\rho}$ and $\boldsymbol{\lambda}$, which are broadcast to all agents for the next iteration. In fact, it is a burden to set a coordination center for the whole network in the real-world cases. A max consensus algorithm is further devised to assign $\boldsymbol{\lambda}$ and $\boldsymbol{\rho}$ to avert a coordination center. The detailed steps of the decentralized algorithm are listed as below.

Algorithm: decentralized dual decomposition algorithm for P_1

Step 1: **Initialize** $k=0$, set $\boldsymbol{\lambda}_i(0)=\boldsymbol{0}$, $\boldsymbol{\rho}_i(0)=\boldsymbol{0}$, $\boldsymbol{s}_i^+(0)=-\infty$, $\boldsymbol{s}_i^-(0)=+\infty$, for all $i=1,2,\ldots,m$

Step 2: **Repeat**

Step 3: **For** $i=1,2,\ldots,m$ **do**

Step 4: $\boldsymbol{\psi}_i(k)=\sum_{j\in N_i} \boldsymbol{\alpha}_{ij}(k)\boldsymbol{\lambda}_j(k)$

Step 5: $\boldsymbol{x}_i(k+1)=\arg\min_{\boldsymbol{x}_i \in X_i} (\boldsymbol{c}_i^{\mathrm{T}} + \boldsymbol{\psi}_i(k)^{\mathrm{T}} \boldsymbol{A}_i)\boldsymbol{x}_i$

Step 6: $\boldsymbol{\phi}_i(k)=\max_{j\in N_i} \{\boldsymbol{\rho}_j(k)\}$

Step 7: $\boldsymbol{s}_i^+(k+1)=\max\{\boldsymbol{s}_i^+(k), \boldsymbol{A}_i\boldsymbol{x}_i(k+1)\}$

Step 8: $\boldsymbol{s}_i^-(k+1)=\min\{\boldsymbol{s}_i^-(k), \boldsymbol{A}_i\boldsymbol{x}_i(k+1)\}$

Step 9: $\boldsymbol{\rho}_i(k+1)=\max\{\boldsymbol{\phi}_i(k), q(\boldsymbol{s}_i^+(k+1)-\boldsymbol{s}_i^-(k+1))\}$

Step 10: $\boldsymbol{\lambda}_i(k+1)=[\boldsymbol{\psi}_i(k)+\boldsymbol{\beta}(k)(\boldsymbol{A}_i\boldsymbol{x}_i(k+1)-(\boldsymbol{b}-\boldsymbol{\rho}_i(k+1))/m]_+$

Step 11: $k=k+1$

Step 12: **Until** $\boldsymbol{x}_1(k), \boldsymbol{x}_2(k), \ldots, \boldsymbol{x}_m(k)$ satisfy the coupling constraints

In *Step 4*, agent i gathers $\boldsymbol{\lambda}_j(k)$ of its connected agent j, and the average $\boldsymbol{\psi}_i(k)$ for $\boldsymbol{\lambda}_j(k)$ is constructed by using the weighted factor $\boldsymbol{\alpha}_{ij}(k)$; and N_i denotes the group of the neighboring agents for agent i. The dual variables of agent i are fixed as $\boldsymbol{\psi}_i(k)$, and the min optimization is performed in *Step 5* to reap a tentative primary solution $\boldsymbol{x}_i(k+1)$. In *Steps 6-9*, each agent refines the coefficient $\boldsymbol{\rho}_i(k+1)$ via the max and min optimizations $\boldsymbol{s}_i^+(k+1)$ and $\boldsymbol{s}_i^-(k+1)$ of components $\boldsymbol{\phi}_i(k)$, respectively. Specifically, *Steps 7* and *8* obtain the worst contributions of $\boldsymbol{A}_i\boldsymbol{x}_i(k+1)$ to the coupling constraints by the tentative solution $\boldsymbol{x}_i(k+1)$, and a tightening coefficient $\boldsymbol{\rho}_i(k+1)$ is derived as the maximum one of $\boldsymbol{\rho}_i(k)$ and $\boldsymbol{\rho}_j(k)$ in *Steps 6* and *9*. In *Step 10*, the dual variables are updated along the gradients via $\boldsymbol{x}_i(k+1)$; $\boldsymbol{\beta}(k)$ denotes the step size whose selection rules can be found in [17] to ensure the convergence property of this algorithm; and $[\cdot]_+$ denotes a projection to the non-negative quadrant with q-dimensions. Note that *Steps 4* and *6* realize the decentralization of the dual decomposition, and each agent does not need to disclose its privacy information, but only the tentative $\boldsymbol{\lambda}_i(k)$ and $\boldsymbol{\rho}_i(k)$.

Note that the decentralized solution converges to the dual optimal solution in finite iterations with the dual subgradient algorithm. Going into detail, the dual variable sequence $\boldsymbol{\lambda}_i(k)$ created by *Step 10* guarantees the convergence to the dual optimal solution. Such a dual finite-time convergence proposition for MILP models is presented in [17]. Since a duality gap for the solutions of the primal and dual programming will exist due to non-convexity, the coupling constraint tightening in P_2 recovers a feasible primal solution using the dual solution of the tightened primal problem [16]. The primal finite-time feasibility property has been proven (see Theorem 1 in [18]). Meanwhile, Theorem 2 in [18] further indicates the optimal performance property. Therefore, the dual decomposition algorithm could derive a decentralized feasible solution for the primal programming that fills tight performance bounds within finite iterations. Interested researchers can refer to [18] for elaborated mathematical proofs on the algorithmic optimal and convergent properties of MILP problems.

C. Remarks on Proposed P&D Approach

In this study, we consider the source-load power uncertainty shown in (1) for multi-agent coordinated scheduling of power systems. Note that there would be uncertainty in the availability of power generation or consumption resources in practice, while the proposed P&D approach is not limited considering certain kinds of uncertainty. In other words, the scheduling model in (1) is general for multiple uncertainty. For instance, when we further consider the $N-k$ output uncertainty of power units [19], which is a typical uncertainty in the availability of generation, we only need to incorporate the uncertainty set of unit outage into $\boldsymbol{u}_i$ and Ω_i, and this does not affect the essential formulation of the mathematical model in (1). Hence, the proposed P&D approach is still applicable under this condition.

It is noteworthy that this paper focuses on how to achieve an effective decentralized solution to a centralized RO problem and ensure that the derived decentralized results are as close as possible to the optimal results of the centralized method. From the perspective of game theory, the relationship of the multi-agent objectives in this paper belongs to the cooperative game. There also exists the condition of multiple agents with conflicting objectives in scheduling. Currently, some effective models or methods have been investigated to address this problem, such as the Stackelberg game model [20] and the Nash equilibrium model, but this topic goes beyond the scope of this work.

IV. Numerical Tests

Three cases of multi-area scheduling are evaluated to verify the superiority and scalability of the proposed P&D approach. Case 1 is a small-scale 2-area 12-bus system with one wind farm integrated into each area and an interconnected tie-line. Case 2 and Case 3 both adopt a 1441-bus system with practical size, which are divided into 4 areas and 8 areas, respectively. The real 1441-bus system contains 130 wind farms, 320 conventional generators, 2057 internal lines, and 23 tie-lines connecting multiple areas with a voltage level of 500 kV. Detailed data of the two test systems are available in [9] and [2], respectively. For each case, the proposed P&D approach (A0), the centralized TSRO method

(A1) [10], and the distributed TSRO method based on ADMM (A2) [4] are used for comparison. The detailed centralized TSRO model before distributed decomposition is formulated in [13]. All TSRO models in compared methods are tackled by the C&CG algorithm with strong duality. Besides, 1000 stochastic scenarios are generated for Case 1 and 20000 for Cases 2 and 3, to check the feasibility of their basic plans, and those scenarios with power balance are recorded. The optimization horizon and the period interval are 24 hours and 1 hour, respectively. Numerical tests are implemented on a laptop under 2.30 GHz i5-CPU and 8 GB RAM, and CPLEX 12.8 solves all MILPs. The optimal results of different cases using various robust scheduling methods are listed in Table I.

TABLE I
OPTIMAL RESULTS OF DIFFERENT CASES

Case	Method	Cost ($)	Iteration	Time (s)	Number of scenarios	Problem scale	
						Number of constraints	Number of variables
Case 1 (2-area 12-bus system)	A1	150243	N/A	4.9	1000	22646	10638
	A0	150280	23	37.6	1000	28126	7352†
	A2	152326	78	1146.5	1000	22694	10710‡
Case 2 (4-area 1441-bus system)	A1	29376412	N/A	100.4	20000	1248106	465184
	A0	29382876	27	95.2	20000	1512704	303374†
	A2	29768991	132	3880.6	20000	1248298	465568‡
Case 3 (8-area 1441-bus system)	A1	29376412	N/A	100.4	20000	1248106	465184
	A0	29381995	30	63.3	20000	1512822	303402†
	A2	29617300	146	3433.7	20000	1248490	465952‡

Note: † represents the total constraints and variables of the models after elimination for all agents; and ‡ represents the total constraints and variables of the decoupled models after ADMM for all agents.

In Case 1, it is noteworthy that the total operation cost of A0 is approximately equal to the referring centralized solution in A1, and the relative optimality gap is 0.025%, whereas the gap for A2 is 1.39%. The dual decomposition algorithm realizes the effective decoupling of the non-convex MILP problem, and the constraint tightening contracts the duality gap that ensures the solution quality. The number of distributed iterations and the total solution time of A2 are 3.4 and 30.5 times of those in A0, respectively. The reason is that the dual decomposition algorithm updates the dual variables in the direction of gradients, which accelerates the convergence of the problem solution. The update of the multipliers and the penalty factors in ADMM depends on the empirical values, resulting in a low convergence speed. In addition, each agent directly solves the MILP-type robust scheduling model after projection, which greatly reduces the computational time for each distributed iteration.

For larger-scale testing systems in Case 2 and Case 3, the optimality gap between A0 and A1 is further reduced. In particular, the optimality gaps for Case 2 and Case 3 are 0.022% and 0.019%, respectively, indicating that the optimal cost is closer to the centralized objective with more agents. Besides, the iteration of A0 increases slowly, hence its computational performance is relatively stable. The solution within minutes in A0 fully meets the scheduling requirements, while the iteration number of A2 is 4.8 times of that by A0, and the total run time even reaches 40-54 times. It can also be observed that the increase of the modeling scale leads to a significant rise in the iterations and the run time of A2, and the hour-level computational time hinders its practical application. Although the total number of constraints after elimination in A0 increases compared with A1 and A2, the dual decomposition further divides the entire problem into smaller local scheduling problems for each agent, hence it will not further pose huge computational challenges to the decentralized implementation by agents. Therefore, A0 enjoys stable computational performance and high scalability in terms of agent number and system size, and these strengths are more prominent compared with A2. Besides, for all cases, the three methods (A0, A1, and A2) derive the optimal power plans that guarantee 100% feasibility to all stochastic scenarios, indicating the robustness of their solution results.

V. CONCLUSION

This letter proposes a novel P&D approach for the robust coordinated scheduling of multiple agents under uncertainty. Numerical tests denote that the proposed P&D approach overcomes the defects of low computational efficiency, slow convergence, and suboptimality in the traditional distributed TSRO method. The merits of the proposed P&D approach in optimality and computational performance are more notable with the enlargement of the system scale and agent quantity. Note that the proposed P&D approach is a universal distributed TSRO method that can be easily extended to numerous coordinated scheduling problems such as multi-area unit commitment, transmission-distribution coordination, energy management of networked microgrids, peer-to-peer trading in prosumers, and integrated energy system dispatch. Future research will consider extending the P&D approach for distributed TSRO methods with binary recourses and endogenous uncertainty. Besides, further research is needed on distributed scheduling of multiple agents with conflicting objectives under uncertainty.

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